

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 1

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 1

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

1. How many real roots does  $3v^2 - 4v - 5 = 0$  have?
2. What is the largest possible area, in square meters, of a triangle with vertices on a circle with a radius of 4 m?
3. In a right triangle with legs measuring 8 m and 4 m, what is the cosecant of the smallest angle?
4. What is the sum of the first fifteen terms of an arithmetic sequence with first term forty and common difference seven?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

1. How many real roots does  $3v^2 - 4v - 5 = 0$  have?
2. What is the largest possible area, in square meters, of a triangle with vertices on a circle with a radius of 4 m?
3. In a right triangle with legs measuring 8 m and 4 m, what is the cosecant of the smallest angle?
4. What is the sum of the first fifteen terms of an arithmetic sequence with first term forty and common difference seven?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 2

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 2

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

5. What are the coordinates, in the form  $(x, y)$ , of the  $y$ -intercept of the line  $9x - 2y = 54$ ?
  
6. Two concentric circles have radii of 7 m and 4 m. What is the length, in meters, of a chord of the larger circle that is tangent to the smaller circle?
  
7. How many positive integers less than 70 are prime?
  
8. Express in simplest radical form:  $\sqrt{7632}$

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

5. What are the coordinates, in the form  $(x, y)$ , of the  $y$ -intercept of the line  $9x - 2y = 54$ ?
  
6. Two concentric circles have radii of 7 m and 4 m. What is the length, in meters, of a chord of the larger circle that is tangent to the smaller circle?
  
7. How many positive integers less than 70 are prime?
  
8. Express in simplest radical form:  $\sqrt{7632}$

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 3

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 3

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

9. Evaluate the matrix product:  $[1 \quad -2 \quad 3] \begin{bmatrix} 4 & 5 \\ -6 & 7 \\ 8 & -9 \end{bmatrix}$

10. In the new casino game of Headers, you pay \$3 to flip five coins. You receive one dollar back for each coin that shows heads. What is the expected value, in dollars rounded to the nearest cent, of your loss? Note that an expected loss would be expressed as a positive number, while an expected gain would be negative.

11. A cube of solid white plastic with edges measuring 7 m is painted blue on five faces and then sliced into cubes that are 1 m on each edge. How many of these smaller cubes have exactly one blue face?

12. Bart's age is the sum of Lisa and Maggie's ages. Five years ago, Bart's age was twice Lisa's age, which was twice Maggie's age. How old is Bart now?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

9. Evaluate the matrix product:  $[1 \quad -2 \quad 3] \begin{bmatrix} 4 & 5 \\ -6 & 7 \\ 8 & -9 \end{bmatrix}$

10. In the new casino game of Headers, you pay \$3 to flip five coins. You receive one dollar back for each coin that shows heads. What is the expected value, in dollars rounded to the nearest cent, of your loss? Note that an expected loss would be expressed as a positive number, while an expected gain would be negative.

11. A cube of solid white plastic with edges measuring 7 m is painted blue on five faces and then sliced into cubes that are 1 m on each edge. How many of these smaller cubes have exactly one blue face?

12. Bart's age is the sum of Lisa and Maggie's ages. Five years ago, Bart's age was twice Lisa's age, which was twice Maggie's age. How old is Bart now?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 4

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 4

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

13. How many months are in  $3\frac{3}{4}$  decades?
14. Express the domain of  $p(q) = \frac{\sqrt{4-q^2}}{q-2}$  in interval notation. Assume the domain and range are both subsets of the real numbers.
15. Two circles with radii of 2 m and 4 m have their centers 7 m apart. A line segment is drawn from one circle to the other, tangent to both. What is the shortest, in meters, such a line segment can be?
16. A data set has a median of 5689. When two elements of 8359 are added to the data set, the new median is 8345. When six additional elements of 2380 are added to the data set, the new median is 4579. When four more elements of 67925 are added to the data set, what is the new median?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

13. How many months are in  $3\frac{3}{4}$  decades?
14. Express the domain of  $p(q) = \frac{\sqrt{4-q^2}}{q-2}$  in interval notation. Assume the domain and range are both subsets of the real numbers.
15. Two circles with radii of 2 m and 4 m have their centers 7 m apart. A line segment is drawn from one circle to the other, tangent to both. What is the shortest, in meters, such a line segment can be?
16. A data set has a median of 5689. When two elements of 8359 are added to the data set, the new median is 8345. When six additional elements of 2380 are added to the data set, the new median is 4579. When four more elements of 67925 are added to the data set, what is the new median?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 5

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 5

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

17. When Mr. E asks his students to find the roots of an equation of the form  $g^2 + Bg + C = 0$ , Hao miscopies the value of B to get roots of -24 and  $-\frac{1}{2}$ , while Iker miscopies the value of C to get roots of -3 and 11. What are the roots of Mr. E's original equation?
18. What is the area, in square meters, of an isosceles triangle with sides measuring 5 m and 13 m?
19. A bag contains four red, seven green, and three blue marbles. If three marbles are chosen randomly, what is the probability that exactly two of them are the same color?
20. What is the missing term in the sequence beginning 4, 5, 12, 13, 36, 35, 108, 71, 324, \_\_\_\_, ...?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

17. When Mr. E asks his students to find the roots of an equation of the form  $g^2 + Bg + C = 0$ , Hao miscopies the value of B to get roots of -24 and  $-\frac{1}{2}$ , while Iker miscopies the value of C to get roots of -3 and 11. What are the roots of Mr. E's original equation?
18. What is the area, in square meters, of an isosceles triangle with sides measuring 5 m and 13 m?
19. A bag contains four red, seven green, and three blue marbles. If three marbles are chosen randomly, what is the probability that exactly two of them are the same color?
20. What is the missing term in the sequence beginning 4, 5, 12, 13, 36, 35, 108, 71, 324, \_\_\_\_, ...?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 6

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 6

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

21. If  $u(v) = (v + 2)^3 \sin(4v)$ , evaluate  $u'(0)$ .
22. Express  $6.4\overline{8}$  as an improper fraction.
23. When two cards are drawn from a standard 52-card deck without replacement, what is the probability that the first card is a red face card (J, Q, or K), and that the second is a Spade?
24. What is the equation of the axis of symmetry of the graph of  $y = 2x^4 - 24x^3 + 109x^2 - 222x + 168$ ?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

21. If  $u(v) = (v + 2)^3 \sin(4v)$ , evaluate  $u'(0)$ .
22. Express  $6.4\overline{8}$  as an improper fraction.
23. When two cards are drawn from a standard 52-card deck without replacement, what is the probability that the first card is a red face card (J, Q, or K), and that the second is a Spade?
24. What is the equation of the axis of symmetry of the graph of  $y = 2x^4 - 24x^3 + 109x^2 - 222x + 168$ ?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 7

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 7

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

25. What are the coordinates, in the form  $(x, y)$ , of the vertex of the parabola  $y = 8x^2 - 9x - 4$ ?

26. In a triangle with sides measuring 4 m, 5 m, and 6 m, what is the length, in meters, of the median to the longest side?

27. In the cryptarithm below, each instance of a letter represents the same digit 0-9, and different letters represent different digits. E.g. if one A represents a 1, all A's represent 1's and B's cannot represent 1's. What is the largest possible value of the five-digit number ABCDE?  
 $AB \times BC = DEA$ ?

28. What is the sum of the positive two-digit even numbers that do not contain a 2?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

25. What are the coordinates, in the form  $(x, y)$ , of the vertex of the parabola  $y = 8x^2 - 9x - 4$ ?

26. In a triangle with sides measuring 4 m, 5 m, and 6 m, what is the length, in meters, of the median to the longest side?

27. In the cryptarithm below, each instance of a letter represents the same digit 0-9, and different letters represent different digits. E.g. if one A represents a 1, all A's represent 1's and B's cannot represent 1's. What is the largest possible value of the five-digit number ABCDE?  
 $AB \times BC = DEA$ ?

28. What is the sum of the positive two-digit even numbers that do not contain a 2?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 8

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 8

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

29. A triangle has two sides measuring 39 m and 56 m, subtending an angle of  $135^\circ$  between them. What is the area, in square meters, of this triangle?

30. Simplify in terms of  $i(= \sqrt{-1})$ :  $i^7(4 - i)^2 + i^{-3}(7 + i)$

31. A prospector needs to visit both the river ( $y - x = 10$ ) and his claim  $(12, -5)$  (in either order) on the way to the town of Origin. If he is currently at  $(9, 15)$ , what is the shortest distance he can travel?

32. A  $3 \times 4 \times 5$  box is completely filled with  $1 \times 1 \times 2$  blocks. After this, a hole is drilled between two opposite vertices. What is the smallest number of blocks that this hole could pass through? Note: passing through only an edge or only a vertex does not count as passing through a block.

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

29. A triangle has two sides measuring 39 m and 56 m, subtending an angle of  $135^\circ$  between them. What is the area, in square meters, of this triangle?

30. Simplify in terms of  $i(= \sqrt{-1})$ :  $i^7(4 - i)^2 + i^{-3}(7 + i)$

31. A prospector needs to visit both the river ( $y - x = 10$ ) and his claim  $(12, -5)$  (in either order) on the way to the town of Origin. If he is currently at  $(9, 15)$ , what is the shortest distance he can travel?

32. A  $3 \times 4 \times 5$  box is completely filled with  $1 \times 1 \times 2$  blocks. After this, a hole is drilled between two opposite vertices. What is the smallest number of blocks that this hole could pass through? Note: passing through only an edge or only a vertex does not count as passing through a block.

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 9

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

## Round 9

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

33. What value(s) of  $f$  satisfy  $8f^2 + 2f - 4 = 0$ ?
34. What is the sum of the 25 smallest positive odd numbers that are not perfect squares?
35. Set R is the set of positive multiples of five less than 69, Set S is the set of multiples of four from 40 to 444 inclusive, and Set T is the set of multiples of three greater than 33. How many elements are in the set  $(T' \cap S) \cup R$ ?
36. What is the area of the convex pentagon with vertices at the points  $(2, -3)$ ,  $(-9, 5)$ ,  $(8, 9)$ ,  $(-6, -6)$ , and  $(-8, 9)$ ?

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

33. What value(s) of  $f$  satisfy  $8f^2 + 2f - 4 = 0$ ?
34. What is the sum of the 25 smallest positive odd numbers that are not perfect squares?
35. Set R is the set of positive multiples of five less than 69, Set S is the set of multiples of four from 40 to 444 inclusive, and Set T is the set of multiples of three greater than 33. How many elements are in the set  $(T' \cap S) \cup R$ ?
36. What is the area of the convex pentagon with vertices at the points  $(2, -3)$ ,  $(-9, 5)$ ,  $(8, 9)$ ,  $(-6, -6)$ , and  $(-8, 9)$ ?

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 10

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

Round 10

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

37. Arrange the letters A-D in order of **descending** value.

$$A = .9705$$

$$B = \frac{2\pi}{7}$$

$$C = \frac{\sqrt{2}}{2}$$

$$D = \frac{8}{9}$$

38. To allow his dog to run around in the park, its owner ties a rope between two trees that are 20 m apart so that there is 30 m of rope between the trees, and runs that rope through the dog's collar. What is the area, in square meters, that the dog can roam?

39. In a nine-element data set of integer test scores from 0 to 100 inclusive, the mean is 49, the median is 74, and the unique mode is 20. What is the largest possible value of the range?

40. Find a solution, in the form  $(h, j, k, m)$ , of the system of equations  $h + j + k + m = -10$ ,  $hj + hk + hm + jk + jm + km = 23$ ,  $hjk + hjm + hmk + jkm = 10$ ,  $hjkm = -24$ .

---

2017 Four-by-Four Competition  
Thursday, February 2nd, 2017

37. Arrange the letters A-D in order of **descending** value.

$$A = .9705$$

$$B = \frac{2\pi}{7}$$

$$C = \frac{\sqrt{2}}{2}$$

$$D = \frac{8}{9}$$

38. To allow his dog to run around in the park, its owner ties a rope between two trees that are 20 m apart so that there is 30 m of rope between the trees, and runs that rope through the dog's collar. What is the area, in square meters, that the dog can roam?

39. In a nine-element data set of integer test scores from 0 to 100 inclusive, the mean is 49, the median is 74, and the unique mode is 20. What is the largest possible value of the range?

40. Find a solution, in the form  $(h, j, k, m)$ , of the system of equations  $h + j + k + m = -10$ ,  $hj + hk + hm + jk + jm + km = 23$ ,  $hjk + hjm + hmk + jkm = 10$ ,  $hjkm = -24$ .