

2016 Four-by-Four Competition  
Thursday, February 4th, 2016

Round 1

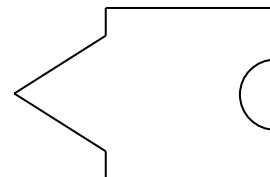
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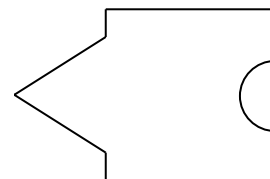
1. When Tom guesses that Anne's number is 240, Anne grins because he's off by exactly 20% of her number. What is the sum of all possible values of Anne's number?
2. When three fair six-sided dice are rolled, what is the probability that they show numbers that sum to 7?
3. The figure below is a square with two opposite sides modified by adding an equilateral triangle and removing a semi-circle. For your answer, draw a modified version of this figure (modifying the other two sides) that could tessellate a plane. Your drawing does not have to be to scale, but the shape must be clear to scorers.
4. What is the missing term of the sequence 2, 3, 4, 9, 32, \_\_\_\_, 8896, ...



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Round 2

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5. Evaluate:  $75060 \div 695$

6. Evaluate:  $\frac{10!}{2^5 \cdot 5!}$

7. A right circular cylinder has a lateral surface area that is half its total surface area, and it has a height of 12 m. What is its volume, in cubic meters?

8. What is the missing term of the harmonic sequence ..., 168, 210, \_\_\_\_, 420, ...?

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## Round 3

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9. What is the perimeter, in meters, of a parallelogram with sides measuring 78 m and 34 m?
10. A triangle is drawn with vertices at three of the endpoints of the axes of the ellipse with equation  $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{9} = 1$ . What is the maximum possible area of such a triangle?
11. The probability of rain is  $\frac{2}{3}$ , the probability that I watch TV is  $\frac{1}{6}$ , and the probability that I play board games is  $\frac{3}{4}$ . If these events are all independent, what is the probability that at most one of these things occurs?
12. What is the shortest distance between the point (85,70) and the line  $3x = 4y$ ?

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13. What is the volume, in cubic meters, of a right circular cone with a base radius of 6 m and a height of 8 m?
14. What number is half the sum of 8925 and the product of 85 and 79?
15. Express the binary numeral  $11000100_2$  in hexadecimal.
16. My mini-Magic deck contains three different blue cards, four different green cards, and five different red cards. After I shuffle the deck, what is the probability that the cards are grouped by color?

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## Round 5

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17. Sunit wishes to create a solution that is 40% sugar, but she only has 12 liters of 28% sugar solution and 18 liters of 64% sugar solution. What is the largest possible volume, in liters, of 40% sugar solution she can create by mixing these two?
18. Quadrilateral ABCD has sides measuring 4 m, 6 m, 8 m, and 9 m. Quadrilateral EFGH is similar to ABCD, and has two sides measuring 8 m and 12 m. What is the smallest possible perimeter, in meters, of quadrilateral EFGH?
19. What is the sum of the positive two-digit odd integers?
20. How many positive integers less than one million are palindromes written using at least one 7?

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21. Evaluate:  $\frac{1}{2} \div \frac{1}{3} \times \frac{1}{4} - \frac{1}{6} \left( \frac{2}{3} \times \frac{3}{4} \right) \div \frac{5}{6}$

22. Using the table to the right, evaluate  
 $j(p^{-1}(m(1))) \times k(p(2)) + m^{-1}(j(k(3)))$

$n$	-2	-1	0	1	2	3
$j(n)$	-1	1	2	-2	0	3
$k(n)$	3	0	-2	1	2	-1
$m(n)$	0	-1	3	2	1	-2
$p(n)$	2	-2	-1	0	3	1

23. Every Platonic Solid has a “dual”, which is a Platonic Solid formed by using the centers of each face of the original solid as the vertices of the new solid. What is the name of the Platonic Solid that is its own dual?

24. Evaluate:  $\binom{22}{13}$

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25. What is the area, in square meters, of a triangle with sides measuring 5 m, 7 m, and 8 m?
26. What is the inverse of the matrix  $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ ?
27. James starts at 695 and colors every 8,040th number on the number line brown. Danielle starts at 5,489 and colors every 9,375th number on the number line white. What is the smallest possible positive difference between a brown number and a white number?
28. What is the value of  $w$  in the solution to the system of equations  $t + u + v + w = 48$ ,  $2t - 3u + 2v - 2w = 82$ , and  $t - 2u + v - w = 56$ ?

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Round 8

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29. What is the product of the six sixth-roots of 64?
30. Evaluate:  $\int_2^5 \frac{1}{n^2+n} dn$
31. What is the sum of all sums that are more likely when three fair six-sided dice are rolled than they are when two dice are rolled? E.g. 18 has a higher probability when three dice are rolled than when two dice are rolled, so it will be part of your sum.
32. My piggy bank contains 26 coins, each of which is either a penny, nickel, dime, or quarter. The total value of the coins is \$2.94, and there are twice as many of the coins with the smallest radius as there are of the coins with the largest radius. How many nickels are there?

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Round 9

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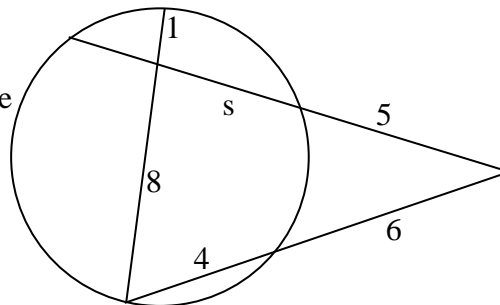
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33. Evaluate:  $\cot \frac{8369\pi}{6}$

34. Express in simplest radical form:  $\sqrt{66825}$

35. The figure to the right shows a circle with two secants and a chord, with most segment lengths labeled in meters. What is the smallest possible value of  $s$ ?



36. Set Q is the set of all multiples of 7 ending in 1, and Set R is the set of all positive three-digit multiples of 3. How many elements are in the set  $R \cap Q'$ ?

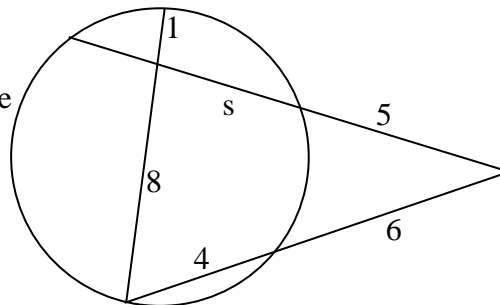
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Round 10

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37. Consider the function  $s(t) = 2t^3 - 3t + 4$  on the interval  $[1,5]$ . What is the value of  $t$  on this interval that satisfies the Mean Value Theorem?
38. What are the coordinates, in the form  $(x, y)$  of the x-intercept of the line through the points  $(532,902)$  and  $(182, -123)$ ?
39. What is the sum of the positive three-digit multiples of four that contain at least one 6?
40. In a triangle with sides measuring 4 m, 6 m, and 9 m, a cevian of length 5 is drawn to the longest side. What is the length of the longer segment into which it divides that side?

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