

2013 Four-by-Four Competition
Thursday, January 31st, 2013

Round 1

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1. What is the sum of the terms of an infinite geometric sequence with first term 2016 and common ratio $\frac{1}{4}$?
2. In how many distinguishable ways can five extra copies of your “sided” house key be arranged on a key ring if they are colored red, orange, yellow, green, and green? You can tell if two “sided” keys are oriented the same way or opposite ways on the key ring.
3. If you can buy K kilograms of corn meal with Q quarters, how many dimes would it take to buy 5 kilograms of corn meal? Express your answer in a reduced fractional form.
4. A pentagon has integer side lengths k, m, n, p, q (not necessarily arranged in that order) when measured in meters. The ratio of m to n is 5, the ratio of p to q is 7, and no two sides are congruent. If $m, n, p, \& q$ are all less than 100, what is the largest possible value of k ?

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Round 2

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5. If $r(s) = s^3 - s - 1$ and $t(u) = u + u^2$, evaluate $r\left(t\left(t(r(1))\right)\right)$.

6. Evaluate: $-9 - (-8)(-7)(-6)^{-2} - 5$

7. What is the area of the convex quadrilateral with vertices at the points $(2,3)$, $(3,-4)$, $(4,5)$, and $(10,-6)$?

8. What is the smallest positive integer greater than 1000 with exactly four distinct prime factors?

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9. What is the perimeter, in meters, of a heptagon with sides measuring 13 m each?
10. In a five-element data set of integer test scores from 0 to 100 inclusive, the mean is 73. What is the smallest possible value of the median?
11. In the system $d + f + g + 4h = 5$, $d + f + 4g + h = 6$, $d + 4f + g + h = 7$, and $4d + f + g + h = 8$, what is the value of d ?
12. Evaluate: $203^3 - 197^3$

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13. Evaluate: $(4096^{2^{-3}})^4$

14. What is the distance between the reflection of the origin across the line $y = 7$ and the reflection of the point $(2,4)$ across the line $x = -5$?

15. Set G is the set of positive three-digit integers that are factors of one billion, and Set H is the set of positive integer multiples of one-hundred less than ten thousand. How many elements are in the set $H \cap G'$?

16. Evaluate: $\int_0^1 me^{2m+1} dm$

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17. What is the tens digit of 12^{12} ?

18. What is the surface area, in square meters, of a right circular cylinder with a base radius of 8 m and a height of 5 m?

19. Evaluate: $\frac{2! \times 4! \times 6! \times 8!}{10!}$

20. If $\sin j = \frac{3}{5}$ and $\cos k = -\frac{5}{13}$, where j and k lie in the same quadrant, evaluate $\tan(j - k)$.

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21. What value(s) of n satisfy $9n - 87 = 6n + 54$?
22. An equilateral triangle is inscribed in a circle, and this circle is then inscribed in a larger equilateral triangle. What is the ratio of the area of the smaller triangle to that of the larger triangle, expressed as a fraction?
23. I have 6567 identical gold coins, but I know that one of them is counterfeit, and very slightly heavier than the others. Fortunately, I have a very accurate balance that I can use to compare the weight of up to 1000 coins with any other group of up to 1000 coins. If I use it wisely, what is the greatest number of times I must be prepared to use the balance to find the counterfeit coin?
24. In what base w will 12345_w equal 3267_{10} ?

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25. When a fair coin is flipped repeatedly, what is the probability that the fifth head occurs on the eighth flip?
26. A regular polygon has interior angles that are five times the size of its exterior angles. How many diagonals does this polygon have?
27. Evaluate in terms of $i = \sqrt{-1}$: $(1 + 2i + 3i^2 + 4i^3)(6i^2 + 7i^3 - 8i^4)$
28. If $a + 2b + 3c = 2a - 3b + c = 3a + 2b - c - 1$, what is the value of $a + b - 2c$?

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Round 8

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29. On the first three days of their road trip, Cherie's family drove 500 miles in 8 hours, 400 miles in 9 hours, and 300 miles in 3 hours. What was their average driving speed, in miles per hour, over those three days?

30. In the recursive sequence defined by $d_0 = 16$ and $d_{f+1} = \left(\frac{d_f}{2}\right)^2 - d_f$, what is the value of d_3 ?

31. The vertices of a regular polygon are lettered consecutively in clockwise order starting with A, B, C, ... If the line through M and F passes through the center, the line through A and what other vertex will also pass through the center?

32. What is the sum of the positive four-digit integers that leave a remainder of 3 when divided by 7?

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Round 9

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33. Express in simplest radical form: $\sqrt[5]{448}$

34. What value(s) of j satisfy $\frac{j-1}{j^2-2j-3} = \frac{j+4}{j^2+3j-2}$?

35. In how many ways can I distribute 20 food pellets into five distinguishable fish tanks so that every tank gets at least one pellet and no two tanks receive the same number of pellets?

36. I have a circular rug that just barely fits in a square room of my house. I'd like to order four custom-made circular rugs to fit in the corners of this room so that they'll barely touch the two walls and the central rug. If my room measures 8 m on a side, what diameter, in meters, should I instruct the rug-maker to make the four rugs?

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37. What is the name of the Platonic Solid on which four equilateral triangles meet at each vertex?

38. Evaluate: $12345.6789 + 2.468 + 86420 + 9.7531$

39. What is the sum of the squares of the roots of $v^3 - 2v^2 + 3v - 4 = 0$?

40. A bag of marbles contains three red and seven blue marbles. My friend and I take turns drawing one marble from the bag, with my friend going first, and neither of us replacing our marbles. What is the probability that I draw the first red marble?

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