2018 Ciphering Time Trials
Solutions

1. **75 is 80% of what number?**

   $\frac{80}{100}x = 75$ becomes $\frac{4}{5}x = 75$, then $x = \frac{5}{4} \cdot 75 = \frac{375}{4}$.

2. **What is the equation, in slope-intercept $(y = mx + b)$ form, of the line through the points $(6, 1)$ and $(3, -11)$?**

   The line has a slope of $\frac{1-(-11)}{6-3} = \frac{12}{3} = 4$, so it's of the form $y = 4x + b$. Substituting the first point gives $1 = 4 \cdot 6 + b$, so that $b = 1 - 24 = -23$, for an answer of $y = 4x - 23$.

3. **What is the prime factorization, in exponential form, of 11880?**

   $$11880 = 11 \cdot 1080 = 11 \cdot 2 \cdot 5 \cdot 108 = 11 \cdot 2 \cdot 5 \cdot 2^2 \cdot 3^2 \cdot 3 = 2^3 \cdot 3^3 \cdot 5 \cdot 11$$

4. **What are the coordinates, in the form (x, y), of the midpoint of the line segment from $(-68, 93)$ to $(78, -71)$?**

   The x-coordinate will be $\frac{-68+78}{2} = \frac{10}{2} = 5$, and the y-coordinate will be $\frac{93+(-71)}{2} = \frac{22}{2} = 11$, for an answer of $(5, 11)$.

5. **Express in simplest radical form: $\sqrt[3]{540}$**

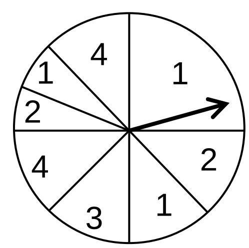
   $\sqrt[3]{540} = \sqrt[3]{27} \cdot \sqrt[3]{20} = 3\sqrt[3]{20}$

6. **What is the range of the median, mode, and mean of the data set {7, 38, 4, 28, 4, 18, 6}?**

   The mode is 4, the median is 7 (probably not necessary), and the mean is $\frac{45+32+28}{7} = \frac{105}{7} = 15$, for a range of $15 - 4 = 11$.

7. **In the number 12,345.67809, what is the product of the digits in the thousands and thousandths places?**

   $2 \cdot 8 = 16$



8. **When the game spinner to the right is spun, what is the probability it points to a 1? Note that the central angles of the sectors are each $90°$, $45°$, or $22.5°$.**

   The probabilities of each 1 are $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$, for a total of $\frac{7}{16}$.

9. **Zilla swims from her lair to The City at a speed of 100 kilometers per hour, then destroys The City. She immediately returns to her lair over the same route, taking four hours for the return trip. If Zilla's average speed for all of her travel was 80 kilometers per hour, how many kilometers was The City from her lair?**

   The time to get to the city is $\frac{d}{100}$, while the time to get back is 4, so her average speed can be written $\frac{2d}{\frac{d}{100}+4} = 80$, which becomes $\frac{200d}{d+400} = 80$, then $\frac{5d}{d+400} = 2$, giving $5d = 2d + 800$, then $3d = 800$, and finally $d = \frac{800}{3}$.

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10. What is the slope of a line perpendicular to the line $3x + 4y = 5$?

The slope of this line is $m = -\frac{A}{B} = -\frac{3}{4}$. The slope of a line perpendicular to this is the negative reciprocal, $-\frac{1}{-\frac{3}{4}} = \frac{4}{3}$.

11. What is the area, in square meters, of a triangle with sides measuring $\sqrt{2}$ m, $\sqrt{5}$ m, and $\sqrt{13}$ m?

These lengths are the hypotenuses of 1x1, 1x2, and 2x3 right triangles, so this triangle might have vertices at $(0, 0)$, $(1, 1)$, and $(2, 3)$, in which case its area would be $2 \times 3 - 1 \times 1 - \frac{1\times1}{2} - \frac{1\times2}{2} - \frac{2\times3}{2} = 6 - 1 - \frac{1}{2} - 1 - 3 = 1 - \frac{1}{2} = \frac{1}{2}$.

12. When eight fair coins are flipped, what is the probability that more of them show heads than tails?

You're just as likely to get more heads as more tails, and it's also possible to get four of each. The probability of the latter is $\frac{8C4}{2^8} = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot2^8} = \frac{7\cdot5}{2^7} = \frac{35}{128}$. This means the probability of the rest is $1 - \frac{35}{128} = \frac{93}{128}$, so that the answer is $\frac{93}{128} \cdot \frac{1}{2} = \frac{93}{256}$.

13. A bag contains 5 red, 9 orange, 9 yellow, 5 green, 4 blue, and 8 purple marbles. If you don't look, how many marbles must you remove to be sure you have at least three of a single color?

There are six different colors, and the worst-case scenario is that you happen to draw two of each color, which is $2 \times 6 = 12$ marbles. At this point, it's impossible not to get a third marble of some color, meaning you must draw $12 + 1 = 13$ marbles to be sure of getting at least three of a single color.

14. What is the circumference, in meters, of a circle circumscribed about a rectangle with sides measuring 6 m by 8 m?

The circle's diameter is the diagonal of the rectangle, and has a length of $\sqrt{6^2 + 8^2} = 2\sqrt{3^2 + 4^2} = 2\sqrt{9 + 16} = 2\sqrt{25} = 2 \cdot 5 = 10$, so the circle's circumference will be 10π.

15. The point $(2, -4)$ is rotated $7470°$ clockwise about the point $(4, 3)$ to point W, then point W is reflected across the line $y = 9$ to point V. What are the coordinates, in the form (x, y), of point V?

$(2, -4)$ is $4 - 2 = 2$ to the left of and $3 - (-4) = 3 + 4 = 7$ below $(4, 3)$. $7470°$ clockwise is equivalent to $7470 - 7200 = 270°$ clockwise, which is equivalent to $90°$ counter-clockwise. This means that the rotated point will be 2 below and 7 to the right of $(4, 3)$, which is $(11, 1)$. This point is $9 - 1 = 8$ below the line $y = 9$, so its reflection will be 8 above the line at $(11, 17)$.

16. What are the coordinates, in the form (x, y), of the center of the conic section with equation $2x^2 - 16x = 3y^2 + 54y$?

Completing the square gives $2(x - 4)^2 = 3(y + 9)^2 + C$, for an answer of $(4, -9)$.

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17. What is the missing term of the harmonic sequence beginning $35, 12,$ __?

This sequence can be considered $12 \cdot 35 = 420$ divided by 12, then divided by 35, and subsequently divided by $35 + 23 = 58$, for an answer of $\frac{420}{58} = \frac{210}{29}$.

18. A cow is tied to an external corner of a closed rectangular barn measuring 8 m by 6 m. If the cow's rope is 12 m long, what is the area the cow can graze, in square meters?

The cow can graze $\frac{3}{4}$ of a circle with a radius of 12, $\frac{1}{4}$ of a circle with a radius of $12 - 8 = 4$, and $\frac{1}{4}$ of a circle with a radius of $12 - 6 = 6$. Our answer is thus $\frac{3}{4} \cdot 12^2\pi + \frac{1}{4} \cdot 4^2\pi + \frac{1}{4} \cdot 6^2\pi = 3 \cdot 6^2\pi + 4\pi + 3^2\pi = 108\pi + 4\pi + 9\pi = 121\pi$.

19. In a solution to the system of equations $8s - 2t + 3u = 5$ and $-4s + t - u = -1$, what is the value of u?

This is two equations in three unknowns, so something special had better happen or there won't be an answer. We can eliminate s by doubling the second equation and adding, getting $8s - 8s - 2t + 2t + 3u - 2u = 5 + (-2)$. Aha, the ts also cancel, giving an answer of $u = 3$.

20. If $k(j) = (2j - 1)^3(j + 1)^4$, evaluate $k'(1)$.

$k'(j) = 3(2j - 1)^2 \cdot 2 \cdot (j + 1)^4 + (2j - 1)^3 \cdot 4(j + 1)^3$, so $k'(1) = 3 \cdot 1^2 \cdot 2 \cdot 2^4 + 1^3 \cdot 4 \cdot 2^3 = 96 + 32 = 128$.

21. A right triangle has a hypotenuse measuring 81 m and a leg measuring 25 m. What is the length, in meters, of the other leg?

The Pythagorean Theorm gives $\sqrt{81^2 - 25^2} = \sqrt{6561 - 625} = \sqrt{5936} = 2\sqrt{1484} = 4\sqrt{371}$.

22. What is the value of the discriminant of $12x^2 - 3x + 9 = 0$?

The discriminant is the interior of the square root in the quadratic equation, $b^2 - 4ac = (-3)^2 - 4 \cdot 12 \cdot 9 = 9 - 432 = -423$.

23. Consider the three types of triangle Scalene, Isosceles, and Equilateral. Of the three, one of them (A) can be considered a more specific form of another (B). As your answer, write the ordered pair (A, B) using words.

A scalene triangle has no congruent sides, an isosceles triangle has at least two congruent sides, and an equilateral triangle has three congruent sides, so the last is a special case of the second, for an answer of (Equilateral, Isosceles).

24. What is the smallest number greater than 1,000 that leaves a remainder of 5 when divided by 8 and a remainder of 1 when divided by 6?

Numbers that leave a remainder of 5 when divided by 8 are 5, 13, 21, 29, … Of these, 13 leaves a remainder of 1 when divided by 6, as will any number that is a multiple of both 8 and 6 more than 13. The least common multiple of 8 and 6 is 24, so any number that is 13 more than a multiple of 24 will fit these constraints. $1000 \div 24 = 41r16$, so $1000 - 16 + 24 = 1008$ is a multiple of 24, so $1008 + 13 = 1021$ is our answer.

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25. What is the sum of the number of vertices, edges, and faces on a regular octahedron?

Its name is because it has 8 faces (each of which is an equilateral triangle). Because each has three sides, one might think that there are $8 \cdot 3 = 24$ edges, but because each edge is shared by two faces, there are really only $24 \div 2 = 12$ edges. Similarly, one might think there are $8 \cdot 3 = 24$ vertices, but because four faces meet at each vertex there are only $24 \div 4 = 6$, for an answer of $8 + 12 + 6 = 26$.

26. How many positive integers are factors of 567?

$567 = 7 \cdot 81 = 3^4 \cdot 7^1$, so there are $(4+1)(1+1) = 5 \cdot 2 = 10$ factors.

27. What is the area of the ellipse with equation $2x^2 + y^2 + 8x - 6y = 8$?

Completing the squares gives $2(x+2)^2 + (y-3)^2 = 8 + 2 \cdot 2^2 + 3^2 = 8 + 8 + 9 = 25$, which becomes $\frac{(x+2)^2}{\frac{25}{2}} + \frac{(y-3)^2}{25} = 1$ in standard form. This means the axes are $\sqrt{25} = 5$ and $\sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$, for an area of $5 \cdot \frac{5\sqrt{2}}{2}\pi = \frac{25\pi\sqrt{2}}{2}$.

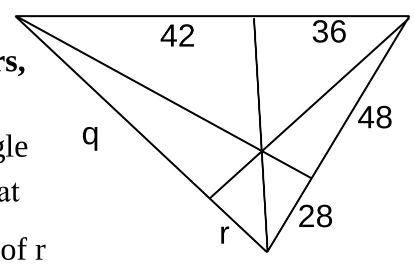
28. What are the coordinates, in the form (x, y), of the vertex of the parabola with equation $y = 3x^2 - 150x + 21$?

The axis of symmetry is $x = -\frac{b}{2a} = -\frac{-150}{2 \cdot 3} = \frac{75}{3} = 25$, so the y-coordinate of the vertex is $y = 3 \cdot 25^2 - 150 \cdot 25 + 21 = 3 \cdot 625 - 3750 + 21 = 1875 - 3750 + 21 = -1875 + 21 = -1854$, for an answer of $(25, -1854)$.

29. What is the cosine of the smallest angle in a right triangle with legs measuring 6 m and 9 m?

The smallest angle will be opposite the shorter leg, which is the 6, so its cosine will be 9 over the hypotenuse, which is $\sqrt{6^2 + 9^2} = 3\sqrt{2^2 + 3^2} = 3\sqrt{4 + 9} = 3\sqrt{13}$, so the answer is $\frac{9}{3\sqrt{13}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$.

30. If all segment lengths in the figure to the right are integers, what is the largest possible value of r?



First, $2 = 78 - 76 < q + r < 78 + 76 = 154$ by the Triangle Inequality. Second, $\frac{r}{q} \cdot \frac{42}{36} \cdot \frac{48}{28} = 1$ by Ceva's Theorem, so that $r = \frac{36 \cdot 28}{42 \cdot 48} q = \frac{3 \cdot 2}{3 \cdot 4} q = \frac{1}{2} q$. The largest possible integer value of r is thus $\left\lfloor \frac{154}{3} \right\rfloor = \frac{153}{3} = 51$.

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