2018 Team Scramble
Thursday, November 8th, 2018

Easier Problems

1. What number is 5397 more than half of 9598?

2. Evaluate: $-50 - 9\big(-43 - (-76)\big) - 53$

3. Evaluate: $59^2 - 41^2$

4. Express $2.3\overline{4}$ as a fraction.

5. What is the name for the place value where the digit 8 appears in the number 49023.71685?

6. Evaluate: $\frac{36}{63} + \frac{7}{56}$

7. What value(s) of b satisfy $4b + 87 = 731$?

8. In which quadrant does the point $(-2357, -1928)$ lie?

9. What is the equation, in slope-intercept form, of the line through the points $(2, -25)$ and $(-5, -4)$?

10. What is the midpoint of the line segment connecting the points $(8, -5)$ and $(-14, -9)$?

11. When the digits of a positive two-digit integer are reversed, the result is a positive two-digit integer that is 54 less than the original number. What is the largest possible value of that original number?

12. What value(s) of w satisfy $4w^2 + 3w - 7 = 0$?

13. What value(s) of m satisfy $\frac{-6m+1}{3m+5} = \frac{12m-9}{-6m+5}$?

14. If $k(j, h) = 9jh - 8j - 7h^2 - \frac{j}{h}$, evaluate $k(4,2)$.

15. When Mr. Brown put an equation of the form $0 = x^2 + Bx + C$ on the board, Sam miscopied the value of B and got roots of -3 and 4. On the same problem, Anthea miscopied the value of C and got roots of 0 and 4. What were the roots of the original problem?

16. What are the coordinates, in the form (x, y), of the left-most x-intercept of the parabola $y = -x^2 - 7x + 9$?

17. What is the shortest distance from the point $(2,4)$ to the line $x - 3y = 4$?

18. If I drive 501 kilometers in 25 hours, then 564 kilometers in 46 hours, what is my average speed in kilometers per hour?

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19. If Pete could roof the shed in 16 hours and Tom could roof it in 24 hours, how many **minutes** would it take them if they worked together?

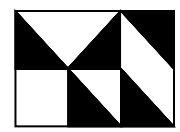
20. What is the area of a right triangle with a hypotenuse measuring 15 m and a leg measuring 12 m?

21. What is the name for a triangle with exactly two congruent sides?

22. A triangle has two sides measuring 84 m and 26 m. If the third side measures c meters, where c is a whole number, what is the largest possible value of c?

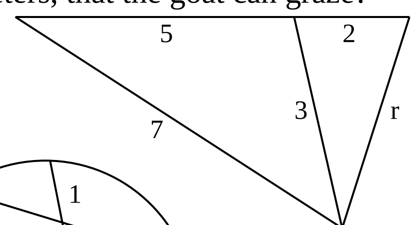
23. What is the circumference, in meters, of a circle circumscribed about a square with a perimeter of 16 m?

24. Two concentric circles contain an annular area of 48π m^2. What is the length, in meters, of a chord of the larger circle that is tangent to the smaller circle?

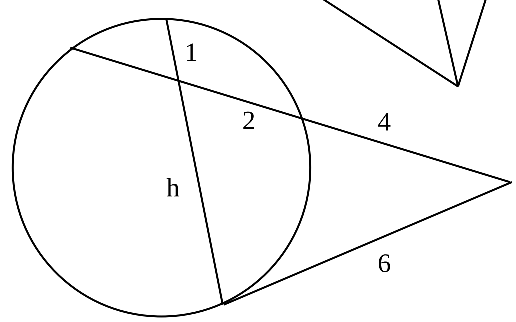


25. What fraction of the 2x3 rectangle to the right is shaded? Assume that shading occurs at 45° angles.

26. A goat is tied to an exterior corner of an enclosed rectangular shelter measuring 2 m by 3 m. If the length of the rope is 4 m, what is the area, in square meters, that the goat can graze?



27. The figure to the right shows a triangle with one cevian drawn, and all segment lengths are given in meters. What is the value of r?



28. A triangle has sides measuring 6 m, 9 m, and 5 m. What is the length, in meters, of the altitude to the shortest side?

29. The figure to the right includes a circle, a chord, a tangent, and a secant, with most line segments labeled in meters. What is the value of h?

30. If $c(d) = 3d^2 - 1$ and $f(g) = 4 + 5g$, evaluate $f^{-1}\big(c(10)\big)$.

31. What is the product of the roots of the polynomial $4h^6 - 2h^7 + 4h^9 + 6h^8 + 6 = 3$?

32. Express the range of $q(r) = 2r^2 + 4r - 3$ in interval notation. Assume the domain and range are both subsets of the real numbers.

33. Express the base 6 numeral 2301_6 as a base 10 numeral.

34. Express the base 10 numeral 6984_{10} as a base 9 numeral.

35. What is the units digit when 697^{231} is evaluated?

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36. What is the sum of the positive integer factors of 960?

37. What is the missing term of the sequence $1, 8, 23, 43, 65, 86, \_\_, \ldots$?

38. What is the missing term of the harmonic sequence 4, 3, ___, …?

39. What is the missing term of the sequence 1, 2, 1, 3, 2, 4, 3, 5, 5, 6, 8, __, …?

40. What is the sum of the counting numbers less than 30?

41. What is the 9^{th} term of a geometric sequence with first term 8 and common ratio 3?

42. When two cards are drawn from a standard 52-card deck, what is the probability that they are of different ranks?

43. A bag contains 8 red marbles and 6 blue marbles. A trusted friend draws two marbles from the bag, looks at them, and tells you they are the same color. What is the probability that they are both blue?

44. I have three different red books, two different blue books, and four different green books. I'd like to put them on a bookshelf so that same-colored books are next to one another. In how many ways can I do this?

45. In a school with 48 boys and 19 girls, 39 students are failing, but 13 girls are passing. How many boys are passing?

46. My fashion teacher has instructed me to bring four socks to school today, consisting of exactly three different colors (exactly two of the socks will be the same color). I forgot to set them aside last night, and because I share a room with my younger sister, I need to grab the socks in the dark before I slip out of my room. Fortunately, I know that my sock drawer contains 1 purple, 4 blue, 6 green, 9 yellow, 4 orange, and 9 red socks. What is the smallest number of socks I can grab in the dark to be sure that I can select four that meet the requirements of my fashion assignment once I get into the light?

47. Evaluate: $\begin{vmatrix} 5 & 6 & 0 \\ 3 & -2 & 1 \\ 0 & 3 & 9 \end{vmatrix}$

48. Set D is {7, 6, 3, 64, 9}, Set E is {50, 70, 9, 2, 4, 1, 5}, and Set F is {6, 67, 7, 3, 5, 9, 8}. Write the set $(F \cap E') \cup D$.

49. Set Q is {-9, -6, 7, 2, 5, -8}, and Set P is {3, 1, 4, 2, 7}. Set N is the largest set that is a subset of both Q and P. Set M is the smallest set that is a superset of both Q and P. What is the product of the number of elements in Set N and the number of elements in Set M?

50. Evaluate: $2 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\cdots}}}$

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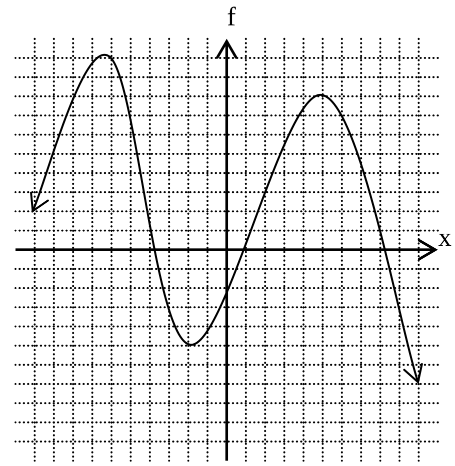
51. If $\sec k = -\frac{5}{4}$, what is the largest possible value of $\cot k$?

52. If h and j are angles in the first quadrant, $\sin h = \frac{3}{5}$, and $\sin j = \frac{5}{13}$, evaluate $\sin(h+j)$.

53. If $\cos m = -\frac{1}{3}$ and m is in the third quadrant, what values can $\sin\frac{m}{2}$ take?



54. Evaluate: $\lim_{n\to 0} \frac{\ln(e-3n)-1}{n}$

55. Approximate $\sqrt{9.2}$ using a first-order differential for $y = \sqrt{x}$ about $x = 9$.

56. Given the graph of f to the right (a fourth-degree polynomial), on what interval(s) is f' decreasing as x increases? Assume that all points of interest occur at the nearest integer value of x. Note the unit grid…

Harder Problems

57. Evaluate: $68^3 - 32^3$

58. Your piggy bank contains 12 half-dollars, 30 quarters, 98 dimes, 4 nickels, and 76 pennies. As a decimal, how many dollars is it all worth?

59. You buy two dozen donuts for \$7.80, and are charged 10% tax on that price. If the baker throws an extra donut into each dozen, how many cents are you paying per donut?

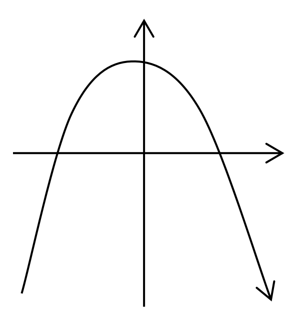
60. Express in simplest radical form: $\sqrt{1920}$

61. Six years ago, JT was three times as old as Wyatt. Five years from now, he'll be twice as old as Wyatt. How old is JT now?

62. Expand and combine like terms: $(a+b-c)(2a-b+3c)$

63. What is the point of intersection, in the form (x, y), of the lines $2x + 6y = 2$ and $y = 6x - 8$?

64. Ignoring scaling, which of the following might be the equation of the parabola shown?



A. $y = +2x^2 + 3x + 97$ B. $y = -2x^2 + 3x + 97$
C. $y = +2x^2 + 3x - 97$ D. $y = -2x^2 + 3x - 97$
E. $y = +2x^2 - 3x + 97$ F. $y = -2x^2 - 3x + 97$
G. $y = +2x^2 - 3x - 97$ H. $y = -2x^2 - 3x - 97$

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65. When the math team goes out for dessert after a contest, they decide to split a Too Huge Chocolate Cake. If there had been one more person, each person would have paid 75 cents less. If there had been two more people than originally, each person would have paid \$1.35 less than they actually did. How many dollars does the cake cost?

66. What value(s) of b can be part of a solution to the system of equations $b + 2c + 3d + 4f = 25$, $2b + c - 3d + 2f = -10$, and $b - c + d - 2f = -14$?

67. What is the most specific name that could be given to a quadrilateral with sides measuring 1 m, 2 m, 3 m, and 4 m (not necessarily in that order)?

68. Two circles have radii of 44 m and 50 m, and their centers are 110 m apart. What is the length, in meters, of an external tangent of those circles?

69. What is the first time after 9:41 AM where the minute hand of a standard 12-hour analog clock is exactly opposite the hour hand? Answer in the form HH:MM:SS to the nearest second, including AM or PM.

70. The faces of a solid ten-inch cube of white plastic are each painted a different non-white color, then the cube is cut into two-inch cubes. How many of these smaller cubes have faces of at least three different colors (potentially including white)?

71. What is the area of an isosceles triangle with sides measuring 8 m and 3 m?

72. What is the length, in meters, of the angle bisector of the largest angle in a triangle with sides measuring 7, 4, and 9 m?

73. A circular rug with a radius of 3 meters is pushed into the corner of a rectangular room so that it touches two walls. A smaller circular rug needs to be designed to fit into the corner so that it touches both walls and the large rug. What should the radius of this smaller rug be, in meters?

74. I invest a million dollars in an account with a 4% interest rate that compounds quarterly. How much money, to the nearest hundredth of a dollar (cent), will be in the account after two years?

75. In the hyperbola with equation $\frac{(y-8)^2}{5} - \frac{(x+2)^2}{16} = 1$, what is the length of a latus rectum?

76. When Mr. E writes an equation of the form $x^3 + Bx^2 + Cx + D = 0$ on the board, Isabella, Sophia, and Mateo miscopy two values each, only getting B, C, and D correct, respectively. When they solve their respective equations, Isabella gets roots of 1, 2, and -10, Sophia gets roots of 1, -2, and 52, and Mateo gets roots of 2, 2, and -18. What were the roots of the original equation?

77. Express the product of 231_5 and 421_5 as a base 5 numeral.

78. What is the least common multiple of 396 and 264?

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79. How many palindromes between 3519 and 8098 are multiples of three and contain the digit 5?

80. What is the largest number less than 1000 that leaves a remainder of six when divided by eight and a remainder of three when divided by nine?

81. How many positive four-digit integers contain exactly one 3 and at least one 4?

82. What is the missing term of the sequence 2, 3, 8, 63, ___, …?

83. Sequence A is arithmetic with first term 8 and common difference 7. Sequence B is geometric with first term 1 and common ratio 2. What is the sum of the numbers less than 10,000 that are in both sequences?

84. What is the sum of the positive integers less than 100 that are not multiples of 8?

85. A recursive sequence is defined with first term $f_1 = 54$ and subsequent terms $f_n = \frac{f_{n-1}}{\sqrt{3}} + \sqrt{3}$. Evaluate f_8, writing your answer in the form $b + c\sqrt{3}$ where b and c are real numbers.

86. On tomorrow's test, the probabilities that Katie, Wily, and Emily get A's are .9, .8, and .7, respectively. What is the probability, as a decimal, that exactly two of them get A's?

87. Jim & Julie play a game in which they take turns rolling a standard six-sided die. The person rolling the die wins if they roll a 1 or 2, or if they roll a number higher than the other person just rolled. What is the probability that the second player wins on their first turn?

88. On the game show Let's Make a Heel, there are one million doors you can choose from, one of which has the cobbler's tools you desire behind them, and the rest of which have nothing behind them. On the show, you get to choose a door, but before you open it, the show's host will open 999,998 doors that she knows to have nothing behind them. At that point, you have the opportunity to open either the door you originally chose or the other remaining unopened door. If you choose to open the door you did not originally choose, what is the probability that your coveted cobbler's tools are behind it?

89. What is the equation, in the form $y = f(x) = g(z)$, of the line through the points $(8, -8, -2)$ and $(5, 1, -1)$?

90. In the data set {9, 48, 5, 56, 9, x, y}, where x and y are counting numbers, the unique mode is greater than the median, which is greater than the mean. What is the largest possible value of $x + y$?

91. Many sets of counting numbers have the properties that their range is less than their unique mode and their mean is greater than their median. Of the many such sets, some of them have the smallest number of elements for such a set. Of these sets, one of them has the smallest mean. Write that set as your answer.

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92. Set B is the set of all positive three-digit multiples of 5, and Set C is the set of integers between 50 and 500 (inclusive) with at least one digit that is a 5. How many elements are in the set $B \cup C$?

93. In a five-digit counting number, the first digit is twice the second digit, the third digit is three times the fourth digit, and the average of the digits is 6. What is the smallest number satisfying these constraints?

94. A Coup deck contains three each of Assassins, Captains, Contessas, Dukes, and Ambassadors. After a particular shuffle, all three Assassins are adjacent, no Captains are, exactly two Contessas are, no Dukes are, and exactly two Ambassadors are. In addition, there is at least one Captain that is adjacent to two Dukes, and at least one Ambassador that is adjacent to two Contessas. Furthermore, the top card in the deck is a Duke and the bottom card is an Ambassador. Finally, all of the Dukes are higher in the deck than all of the Assassins. Given all this, how many distinguishable arrangements of cards are possible?

95. In how many points does the graph of $y = e^x - 2$ intersect $y = \cos(8x)$?

96. Simplify $\sin 2j \cot j - \sec j \cos 2j$ by writing it using the fewest trigonometric functions. Once you have done this, rewrite it by rewriting csc , sec , and cot in terms of sin , cos , and tan .

97. What is the area, in square meters, of a triangle with sides measuring $\sqrt{5}$ m, $\sqrt{20}$ m, and $\sqrt{41}$ m?

98. If $h(k) = \frac{(2k-1)^3}{(2-k)^2}$, evaluate $\frac{dh}{dk}$ when $k = 1$.

99. What is the largest possible area of a rectangle inscribed in an ellipse with equation $4x^2 + y^2 = 4$?

100. Evaluate: $\int_2^5 h\sqrt{h-1}\,dh$