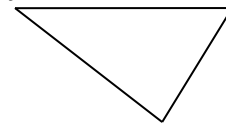


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**Easier Problems**

1. What is the remainder when 2409 is divided by 16?
2. Evaluate:  $8.67 \times 9.1$
3. You have five favorite books, eight favorite stuffed animals, and four favorite beverages, but your dad says you can only bring one book and one stuffed animal, and that you must fill each of two differently-sized water bottles with a different beverage. How many different combinations of favorites can you bring on the trip?
4. Express  $\overline{.84}$  as a reduced fraction.
5. If it is currently 4:46:57 PM, what time will it be in 1000 minutes? Answer in HH:MM:SS format including AM or PM.
6. What is the solution, in the form  $(d, f, g)$ , of the system of equations  $2d + f + g = 12$ ,  $d + 2f + g = 10$ , and  $d + f + 2g = 6$ ?
7. Isaiah can build a cabinet in 8 hours, and Jessica can do it in 6 hours. To the nearest minute, how long would it take them to build the cabinet if they work together?
8. On average, nine chickens take three days to lay 99 eggs. To the nearest day, how many days would we expect it to take for three chickens to lay 79 eggs?
9. Harry sees the Snitch 90 m away, and the chase is immediately on! If the Snitch flies directly away from Harry at 22 meters per second (mps) and Harry flies after it at 28 mps, how many seconds will it take Harry to catch the Snitch?
10. What is the equation of the axis of symmetry of the parabola  $y = -4(x + 3)^2 - 1$ ?
11. What are the coordinates, in the form  $(x, y)$  of the vertex of the parabola  $y = -5x^2 - 9x$ ?
12. What are the coordinates, in the form  $(x, y)$ , of the rightmost  $x$ -intercept of  $y = 48x^2 + 60x + 12$ ?
13. What is the most specific geometric description you can give for the figure to the right?
14. Consider a quadrilateral with three sides measuring 83 m, 25 m, and 8 m. If the fourth side of the quadrilateral is a whole number of meters, what is the product of its longest possible length and its shortest possible length?
15. Consider a right circular cone with a base radius of 3 m and a height of 8 m. Now imagine that it is sliced by two vertical planes through the vertex so that the base of two of the resulting pieces is now a sector of the original circle with a central angle of  $60^\circ$ . What is the volume, in cubic meters, of one of these two pieces?



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16. A quadrilateral with sides measuring 1 m, 2 m, 3 m, and 4 m is similar to a quadrilateral with sides measuring 2 m, 4 m, 6 m, and 8 m. If the area of the smaller quadrilateral is  $3 \text{ m}^2$ , what is the area, in meters, of the larger quadrilateral?
17. A triangle has sides measuring 25 m, 68 m, and 80 m. The angle bisector of the smallest angle is drawn to the opposite side, dividing it into two segments. What is the length, in meters, of the shorter of the two segments?
18. A folding screen with two panels that are each four feet wide is used to create a secluded space in the corner of a rectangular room. What is the maximum possible area, in square feet, of the floor of this space?
19. What is the name of the conic section with equation  $\frac{(x+1)^2}{3} - \frac{(y-4)^2}{5} = 1$ ?
20. Evaluate:  $\log_{32} \frac{1}{128}$
21. How many distinguishable functions are there that map the domain  $\{-4, 4, -7\}$  to the range  $\{-6, -4, 7, 0\}$ ?
22. If  $\log_5 2 = a$  and  $\log_3 5 = b$ , express  $\log 3$  in terms of  $a$  and  $b$ ?
23. Simplify:  $\frac{14t^3 - 11t^2 + 33t - 54}{7t - 9}$
24. Express the hexadecimal number  $E9A$  in base four.
25. List all of the following numbers that are divisible by six:  
7150, 82, 778, 8438, 517, 24, 189, 9879, 369, 40, 280, 3738
26. What is the 37<sup>th</sup> term of an arithmetic sequence with a first term of 234 and a common difference of -96?
27. What is the 5<sup>th</sup> term of a harmonic sequence with first term 8 and second term 6?
28. What is the missing term of the sequence 1, 1, 4, 8, 37, \_\_\_\_\_, ...?
29. What is the sum of the perfect squares less than 100?
30. When 8 fair coins are flipped, what is the probability that exactly six of them show heads?
31. What is the equation of the line through the points (1,2,3) and (2,4,6)? Express your answer in the form  $x = g(y) = h(z)$ .
32. What is the median of the data set  $\{5, 0, 3, 6, 7, \}$ ?
33. Set  $F$  is the set of all positive multiples of 4 less than 1000, and Set  $G$  is the set of multiples of 7 greater than 100. How many elements are in the set  $F \cap G'$ ?

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$$\begin{array}{rcccl} \boxed{A} & \times & \boxed{B} & = & \boxed{12} \\ & & & & \\ & x & + & & \\ \boxed{C} & + & \boxed{D} & = & \boxed{15} \\ & = & = & & \\ \boxed{18} & & \boxed{13} & & \end{array}$$

34. In the cross-math puzzle on the right, A-D are distinct digits (1-9) satisfying the four equations (two across, two down). What is the product of A, B, C, and D?

35. Evaluate:  $\csc \frac{4\pi}{3}$

36. What is the area, in square meters, of a triangle with sides measuring 7 m, 4 m, and 6 m?

37. Express the complex number  $(-8, 8)$  in  $re^{i\theta}$  form, where  $0 \leq \theta < 2\pi$ .

38. What is the value of  $y$  at the local maximum of the function  $y(x) = 2x^3 + 10x^2 - 16x + 1$ ?

39. What is the average value of the function  $y(x) = 2x^3 - x + 4$  on the interval  $[3,5]$ ?

### Harder Problems

40. Evaluate:  $\frac{16! \cdot 10! \cdot 6!}{9! \cdot 5! \cdot 12! \cdot 9! \cdot 5!}$

41. Simplify by rationalizing the denominator:  $\frac{12}{\sqrt[3]{4} - \sqrt[3]{2} + 1}$

42. Because you have a 25%-off coupon, you go to a new restaurant and buy two dinners for \$27.85 each, four appetizers for \$7.85 each, and a flight of five artisanal root beers for \$12.90. You're aware of the 10% sales tax and the 20% automatic gratuity (tip), but are surprised when the bill is higher than you were expecting. You ask your server, who explains that while the sales tax is calculated based on the discounted subtotal, the gratuity is calculated based on the pre-discount subtotal. How much is your bill, in dollars rounded to the nearest hundredth (cent)?

43. What value(s) of  $g$  satisfy  $7g^2 + 8g - 12 = 0$ ?

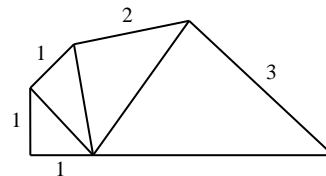
44. Ron, Peter, and Siddhartha walk at constant speeds of 3 mph, 4 mph, and 5 mph, respectively. They decide to have an interesting "race" around a quarter-mile track. They all start at the same point facing in the same direction and begin walking. From then on, any time one of them ends up in the same place as another, the slower of the two must reverse direction. The "race" will continue until the first time that a specific pair is in the same place for the second time. I.e. if Ron & Peter meet up at position C, then later meet up at position B (which might be the same as C), the "race" is over. When the "race" ends, the winner is the person who is not in the same place as the other two. How long will the race take, to the nearest minute?

45. The line through the points  $(-4, -1)$  and  $(6, 2)$  is perpendicular to the line through the points  $(4, -2)$  and  $(-6, v)$ . What is the value of  $v$ ?

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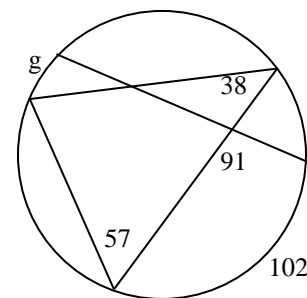
46. When the digits of a positive two-digit integer are reversed to form a new positive two-digit integer, the resulting number differs from one-third of the original number by less than one. What is the smallest possible value of the original number?
47. My piggy bank contains 46 coins worth a total of \$9.05. If the only coins that may be present are pennies, nickels, dimes, and quarters, what is the maximum number of dimes that could be in the piggy bank?
48. A field contains humans, horses, and giant spiders! You see 89 heads, 294 legs, and 226 eyes (you think giant spiders have 8 legs, 8 eyes, and no head). How many giant spiders are there?
49. If  $(r, s, t, u)$  is a solution to the system of equations  $-9r - 7s + 6t - 9u = -33$ ,  $3r - 5s - 2t - 5u = -32$ , and  $-6r + 4s + 4t - u = 14$ , what is the value of  $u$ ?
50. What is one solution, in the form  $(n, p, q)$ , of the system of equations  $n + p + q = 7$ ,  $np + pq + nq = -42$ , and  $npq = -216$ ?
51. A cowboy must move his herd from his current camp at the origin, travel to the river (with equation  $y = x + 2$ ) for water, and proceed to his next camp at  $(2, -2)$ . What is the minimum distance he can travel?

52. A gymnast launches himself from a springboard at a speed of 20 meters per second at an angle of  $30^\circ$  to the horizontal. How far away will he land, to the nearest meter, assuming the ground is horizontal?



53. Consider the figure to the upper right composed of right triangles. Each of the right triangles has a leg whose length in meters is a term from the Fibonacci Sequence, and whose other leg is the hypotenuse of the triangle whose leg is the previous Fibonacci term. Because the first triangle has no prior triangle to use for a leg, both of its legs are 1 meter. If this sequence is continued, what is the area, in square meters, of the sixth triangle?
54. A plane is to be tessellated with a pattern that includes regular dodecagons. What is the minimum number of different types of regular polygons that must be used in such a pattern, including the dodecagon?

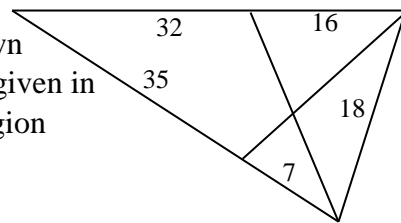
55. The smallest possible circle is circumscribed about two tangent congruent circles with radii of 12 m. What is the side length, in meters, of the largest square that can be “inscribed” outside the two congruent circles but inside the circumscribed circle?



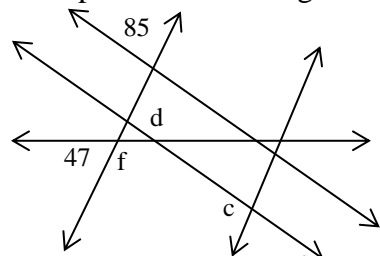
56. The figure to the right includes a circle and many chords, with some angle and arc measures given in degrees. What is the value of  $g$ ?

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57. The figure to the right shows a triangle with two cevians drawn through an interior point, with all perimeter segment lengths given in meters. What is the area, in square meters, of the smallest region created by these cevians?



58. The figure to the lower right shows five lines, two pairs of which are parallel. If all angle measures are given in degrees, what is the value of  $f + d + c$ ?



59. A circular rug fits exactly into a 6-m square room. I'd like to buy a smaller circular rug that will fit exactly into a corner of the room, touching two walls and the large rug. What radius, in meters, should I instruct the rug-maker to make the smaller rug?

60. A torus has an outer radius of 71 m and an inner radius of 35 m. What is the length in meters of the longest line segment that does not extend outside the torus?

61. What is the smallest number of sides a regular polygon can have if the number of diagonals that can be drawn in it is greater than the measure, in degrees, of one of its interior angles?

62. What is the largest number of regions into which a plane can be divided by a quadrilateral, an ellipse, and two lines?

63. A cube of blue plastic is painted green, then cut into 38 smaller cubes. How many of these smaller cubes have exactly two green sides?

64. What is the length of the latus rectum of  $y = 4x^2 - 3x + 2$ ?

65. Chris measures 589.6 kg of Dixium at 12:01 AM on January 1st. If Dixium has a half-life of 137 minutes, what is the first day (month & day) on which Chris will have less than  $1 \mu\text{g}$  of Dixium?

66. When one cow is corralled in a 1 m square corral with 2-cm tall grass, it takes it one day to eat the grass down to the ground. When two cows are corralled in a 2-m square corral with 1-cm tall grass, it takes them two days to eat the grass down to the ground. How many days would it take four cows corralled in a 3-m square corral with 5-cm tall grass to eat the grass down to the ground?

67. What is the largest real value of  $v$  satisfying  $3v^2 + 3vw + 3w^2 = 4$ ?

68. What is the largest possible area of a triangle with vertices on the parabola  $y = -x^2 + 16x - 48$  and with non-negative  $x$  &  $y$  coordinates?

69. Express the product of the base nine numbers  $486_9$  and  $687_9$  in base nine.

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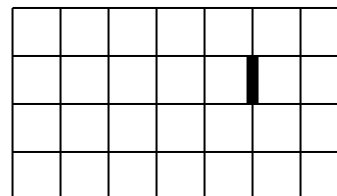
70. What is the smallest number greater than 1000 that is divisible by 9 but leaves a remainder of 8 when divided by 11?
71. How many positive integers are factors of 14040?
72. How many positive integers are factors of both 5268 and 18438?
73. John writes the number 1 and every 54<sup>th</sup> counting number after that (55, 109, etc.). Jane writes the number 47 and every 72<sup>nd</sup> counting number after that (121, 195, etc.). What is the smallest positive difference between a number on John's list and a number on Jane's list?
74. My favorite number is a four-digit counting number with the interesting property that the last two digits form a two-digit counting number that is twice the two-digit counting number formed by the first two digits. In addition, the central two digits form a two-digit counting number that is three times the two-digit counting number formed by the first two digits. What is the largest number that could be my favorite number?
75. What is the tens digit of  $678^{90}$ ?
76. What is the sum of the first 99 terms of an arithmetic sequence with first term -53 and common difference 25?
77. What is the sum of the natural numbers from 78 to 516 inclusive?
78. Sequence U is an arithmetic sequence with first term 64 and common difference 28. Sequence T is a geometric sequence with first term 9 and common ratio 4. How many numbers less than 1000 are common both sequences?
79. Bag S contains 4 purple marbles and 3 blue marbles. Bag R contains 4 purple marbles and 1 blue marble. Two marbles are drawn randomly from Bag S and placed in Bag R. What is the probability that a marble then drawn from Bag R is blue?
80. Your trusted friend James is dealt two cards from a standard 52-card deck. He confides that he does not have two face cards. What is the probability that he has a pair?
81. Your keyring has six keys on it, each of which is one-sided (has a flat side and a bumpy side). If two of the keys are identical, how many distinguishable arrangements are possible?
82. At the Puzzlebrary, there is a shelf of decorative books. There are eight green books, five yellow books, three orange books, and one red book. Five of the green books, two of the yellow books, and one orange book do not have writing on their spines, so they could be placed on the shelf in two possible orientations. Of these blank-spined books, three of the green ones have had their front and back covers glued together to form one giant book, and the two yellow ones have been similarly joined. In how many ways could all of these books be placed next to one another on the shelf if books of the same color must be kept together?

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83. When some people are surveyed about cookie ingredients, 195 say they like chocolate chips, 175 say they like nuts, and 183 say they like coconut. As it turns out, 61 of them like both chocolate chips and coconut, 49 like both nuts and coconut, and 64 of them like both chocolate chips and nuts. If 9 of them like all three ingredients and 4 people like none of them, how many people were surveyed?

84. Sai and Talia play a game where they take turns drawing cards (without replacement) from a standard 52-card deck. If the person who draws the first 9 wins, what is the probability that the first player wins on their second turn (the third turn of the game)?

85. The figure to the right shows an array of unit squares, with one edge thickened. How many paths from the upper left corner to the lower right corner along the gridlines pass through this edge, do not use any edge twice, and have a length of 13?



86. In one phase of a new board game, four tiles are placed face down. It is known that two of the tiles have 0 on their faces, while the other two have 1000 on their faces. You place a cube on top of a tile, then your opponent looks at the **other** three tiles, and must flip exactly one tile with a 0 on its face over so that you can see the 0. At this point, you are allowed to take the tile with your cube on it or either of the other two face-down tiles. If your goal is to get a tile with 1000 on its face and you choose wisely, what is the probability that you achieve your goal?

87. A bubblegum machine has 21 red gumballs, 7 purple gumballs, 37 orange gumballs, 6 blue gumballs, 89 yellow gumballs, and 8 green gumballs. My four children all want gumballs, but two of them are not willing to chew the same color as anyone else. How many gumballs must I be willing to buy in order to be certain I that I can distribute a gumball to each of my children according to their demands?

88. What is the equation of the plane through the points  $(3,1,2)$ ,  $(8,2,1)$ , and  $(1,0,2)$ ? Express your answer in the form  $Ax + By + Cz = D$ , where  $A > 0$  and  $A$ ,  $B$ , and  $C$  are collectively relatively prime integers.

89. What is the shortest distance between the lines  $\frac{x-7}{3} = \frac{y}{2} = \frac{z-9}{4}$  and  $x - 2 = \frac{y+1}{2} = \frac{z+6}{3}$ ?

90. In a data set of seven integer test scores from 0 to 100 inclusive, the unique mode is 90, the median is 55, and the range is 45. What is the smallest possible value of the mean?

91. In the set of integers  $\{31, 6, 130, 93, 605, 99, x, y, z\}$ , the median is less than the mode, which is less than the mean. What is the smallest possible value of  $x + y + z$ ?

92. The average grade in Professor Slughorn's Potions class is 45, although the average among Muggleborns is 51. If his class is 80% purebloods (thus 20% Muggleborns), what is the average grade among the purebloods?

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93. In the cryptarithm below, each instance of a letter represents the same digit (0-9), and different letters represent different digits. E.g. if one A is a 1, all A's are 1's and B cannot be 1. What is the smallest possible value of the five-digit number ABCDE? Note: numbers cannot have zero as their leading digit.

$$\begin{array}{r} AB \\ -BC \\ \hline DE \end{array}$$

94. A Crime has been committed, and the Usual Suspects are brought in for “questioning”, in which they each make one statement.

A: I didn't do it!

B: C&D did it!

C: D didn't do it.

D: Neither A nor E did it.

E: I (Person E) did it.

Forensics has determined that two people worked together on the Crime (and that they're among A-E), and the polygrapher says that there were exactly two lies told, although he's forgotten which statements those were. List all suspects (by letter, A-E) that you can be certain are guilty of the Crime.

95. In the land of Knights & Knaves, Knights always tell the truth and Knaves always lie. You meet a group of six residents who make the following statements:

A: There are exactly two Knights here.

B: There are at least two Knaves here.

C: At least one of A&B are Knights.

D: There is an even number of Knaves here.

E: There are at most four Knights here.

F: At least one of C & D are Knaves.

List all residents (by letter, A-F) that you can be certain are Knights.

96. Evaluate in radians:  $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$

97. A five-meter ladder is on level ground leaning against a vertical wall, when suddenly its feet begin to slide away from the wall at a constant speed of 5 meters per second. At the moment the feet are 4 meters from the wall, how fast is the top of the ladder sliding down the wall, in meters per second?

98. Evaluate:  $\int_{-1}^2 3(4j)^5 dj$

99. Evaluate:  $\int_4^7 4h\sqrt{h-3} dh$

100. What is the volume of the solid generated when the area between the graphs of  $y = x$  and  $y = x^2 - 6$  is rotated about the line  $y = 4$ ?