

2015 Team Scramble
Thursday, November 5th, 2015

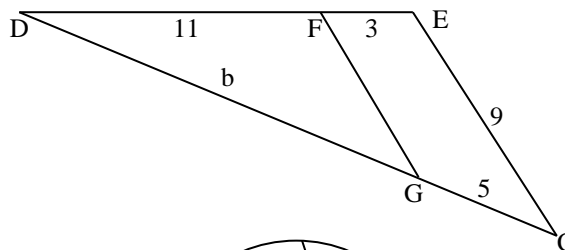
Easy Problems (1-53)

1. Evaluate: $57908 - 7039$
2. What number is 3469 less than three times 8523?
3. Evaluate: $-2(-6 - 8(-9) - 7) - (-3)(-4)(-5) - 8(-9)$
4. Express 436,890,789.45 in scientific notation rounded to four significant figures.
5. Evaluate: $\frac{27^5 \times 3^6}{9^4 \times 81^3}$
6. Simplify by rationalizing the denominator: $\frac{18}{5 + \sqrt{19}}$
7. What is the sum of the number of vertices on an icosahedron, the number of yards in a mile, and the number of hours in a week?
8. Arrange the variables A , B , C , and D in **descending** order.
 $A = 7890 + 234$ $B = 8479790 \div 34$ $C = 8!$ $D = 39 \times 13 + 5 \times 27$
9. Simplify: $f + 2(3f - 4) + (5f + 6)(7 - 8f)$
10. What value(s) of c satisfy $2(3c + 4) + 5(6 - c) = 123$?
11. What value(s) of g satisfy $3g^2 - 7g + 12 = 18$?
12. Miss Higgy and Hermit simultaneously see one another when they are 180 m apart. If Miss Higgy runs towards Hermit at 32 m/s, and Hermit runs away at 23 m/s, how many seconds will it take Miss Higgy to catch Hermit?
13. What are the coordinates, in the form (x, y) , of the x-intercept(s) of the line $4x - 7y = 112$?
14. What are the coordinates, in the form (x, y) , of the midpoint of the line segment from $(81, 234)$ to $(-123, 98)$?
15. What is the equation of the axis of symmetry of a parabola of the form $y = ax^2 + bx + c$ with a vertex at $(4, -7)$ and passing through the point $(-5, 6)$?
16. What are the coordinates, in the form (x, y) , of the vertex of the parabola with equation $x = 3y^2 + 42y - 795$?
17. What are the coordinates, in the form (x, y) , of the y-intercept(s) of the parabola with equation $y = 2x^2 - 7x - 13$?
18. A pasture contains cows, chickens, and farmhands (humans). If there are a total of 100 heads, 320 feet, and 40 hands, how many chickens are there?

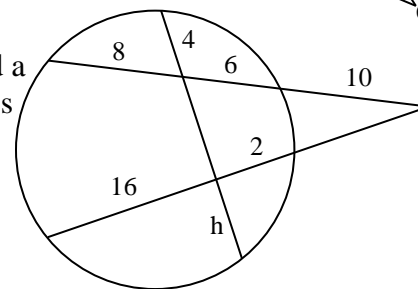
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19. What is the area, in square meters, of a right triangle with a leg measuring 17 m and a hypotenuse measuring 27 m?
20. What is the perimeter, in meters in simplest radical form, of an equilateral triangle with an area of 6 m^2 ?
21. A triangle has two sides measuring 7890 m and 245 m. What is the shortest possible integer number of meters that could be the length of its third side?
22. A rhombus has sides measuring 8 m. If one of the diagonals has a length of 8 m, what is the length, in meters, of its other diagonal?
23. What is the name for a polygon with five sides?
24. What is the volume, in cubic meters, of a cube with a surface area of 294 m^2 ?

25. In $\triangle CDE$ in the figure to the right, $\overline{FG} \parallel \overline{EC}$, and the measures of many line segments are given in meters. What is the value of b ?



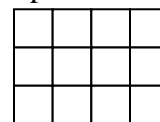
26. The figure to the right shows a circle with two secants and a chord, with many line segments labeled in meters. What is the value of h ?



27. A triangle has sides measuring 9 m, 11 m, and 14 m. What is the length, in meters, of the altitude to its shortest side?
28. How many diagonals can be drawn in a convex nonagon?
29. Evaluate: $9i^7 - 6i^5 + 4i^4 - 3i^2 - 11i + 23$
30. Evaluate: $\log_9 25 \times \log_5 27$
31. If $k(m) = 3m - 4$ and $n(p) = \frac{840}{p}$, evaluate $k\left(n\left(k\left(n(280)\right)\right)\right)$.
32. When $\left(2x - \frac{3}{x}\right)^5$ is expanded and like terms are combined, what is the coefficient of the x term?
33. Express the base 6 number 5235_6 as a base 10 number.
34. Express the base 10 number 6792_{10} as a base 8 number.

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35. When Mrs. Math arranges her students into teams of 3, there are only two people for the last team. When she arranges them into teams of 7, there are also two people for the last team. When she arranges them into teams of 10, there are only three people for the last team. What is the smallest number of students that could be in her class?
36. How many positive integers are factors of 834?
37. What is the sum of the positive integer factors of 180?
38. What is the 23rd term of an arithmetic sequence with first term 4579 and common difference 35?
39. What is the fifth term of a geometric sequence with first term 729 and common ratio $\frac{2}{3}$?
40. What is the next term of the sequence beginning 1, 1, 1, 3, 5, 9, 17, 31, 57, 105, 193?
41. What is the sum of the positive integers less than 32?
42. The probability that I eat popcorn tonight is $\frac{3}{5}$, and the probability that I watch a movie tonight is $\frac{2}{7}$. If these events are independent, what is the probability that I eat popcorn but do not watch a movie?
43. In how many ways can the letters in the word "TATTLETALE" be arranged?
44. When 85 people were surveyed, 48 liked Thing 1 and 76 liked Thing 2. If 2 people liked neither, how many liked both?
45. At a new casino, they charge \$10 for you to roll a standard six-sided die one time. For a 2, 3, 4, 5, or 6 they'll pay you a number of dollars equal to the square of the number you rolled. For a 1, however, you get nothing. What is the expected value of your profit (positive) or loss (negative) if you play this game one time?
46. What is the mean of the data set {5,93,5,7,10}?
47. Set K is the set of all positive two-digit multiples of 4 and Set T is the set of multiples of 3 greater than 50. How many elements does the set $K \cap T'$ have?
48. How many squares of any size are in the array of unit squares to the right?
49. If $\sin u = \frac{1}{6}$ and u is an angle in the second quadrant, what is the value of $\cos 2u$?
50. Evaluate **in radians**: $\cos^{-1}\left(-\frac{1}{2}\right)$
51. Express $12e^{\frac{\pi i}{3}}$ in standard form.
52. If $s(t) = (t + 1)^2(t - 1)^3$, evaluate $s'(2)$.



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53. Evaluate: $\int_2^4 n(n-1)dn$

Hard Problems (54-100)

54. Evaluate: 348×679

55. Evaluate: $39^3 + 41^3$

56. Express $1.\overline{234}$ as a fraction.

57. Hermione and Ivan both bid on a job building a brick wall. Hermione knows she could do it in 12 hours, and Ivan knows he could do it in 16 hours. The customer decides to hire them both, and it turns out that they work together so well that they lay a total of 60 more bricks each hour than they would if they were working simultaneously but separately. If the job requires 4800 bricks, how many **minutes** (to the nearest minute) does it take the two of them to finish the job?

58. Brooks bikes to work at a speed of 25 km per hour, and later bikes home along the same route at a speed of 20 km per hour. What is his average speed, in kilometers per hour, for the entire roundtrip?

59. What is the shortest distance from the point $(38, -9)$ to the line $y = -\frac{4}{3}x + 5$?

60. The point $(-68, 94)$ is rotated 2790° clockwise about the point $(341, 89)$ to point J, which is then reflected across the line $x + y = 249$ to point K, which is then rotated 5760° counter-clockwise about the point $(16, -858)$ to point L. What are the coordinates, in the form (x, y) of point L?

61. When the digits of a positive four-digit integer are reversed, a new positive four-digit integer is created, and the positive difference between the two is M. How many possible values might M have?

62. If you can buy L liters of pudding for D dimes, how many quarters would be needed to buy 40 liters of pudding?

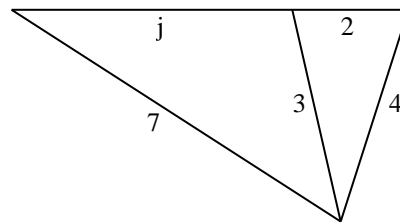
63. Express the solution to the system of equations $m + n + p = 11$, $m + n + q = -8$, $m + p + q = 24$, and $n + p + q = -12$ in the form (m, n, p, q) .

64. What is the area, in square meters, of an isosceles triangle with sides measuring 22 m and 61 m?

65. What is the minimum perimeter, in meters, of a parallelogram with two sides measuring 24 m and an area of 912 m^2 ?

66. What is the volume, in cubic meters, of an octahedron with edges measuring 8 m?

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67. The figure to the right shows a triangle with a cevian, with all line segments labeled in meters. What is the value of j ?
68. A rectangle has sides measuring 2 m and 3 m. What is the largest area, in square meters, that can be covered by two non-overlapping circles that do not extend outside the rectangle? The circles do not necessarily have to be congruent.
69. Two circles with radii of 13 m and 16 m have their centers 30 meters apart. What is the length, in meters, of the shortest line segment that can be drawn tangent to both circles?
70. A cow is tied to the exterior of the 60° corner of a barn in the shape of a 30-60-90 triangle with a short leg measuring 12 m. If the rope has a length of 28 m, what is the area, in square meters, of the grass that the cow can graze? Note: the barn is closed; the cow cannot go inside.
71. In how many distinguishable ways can congruent rectangles with sides measuring 1 m and 2 m be used to tile a square with sides measuring 4 m? Note that the square could be rotated.
72. A right rectangular prism made from black plastic has edges measuring 12 m, 4.5 m, 9 m. It is painted blue on all sides, then cut into the smallest possible number of congruent cubes. How many of the resulting cubes have more black faces than blue faces?
73. What are the coordinates, in the form (x, y) of the upper focus of the conic section with equation $-4x^2 + 24x + 9y^2 + 18y = 243$?
74. What is the area of the ellipse with equation $x^2 + 10x + 4y^2 - 16y = 359$?
75. What value(s) of j satisfy $3^{2j} + 729 = 3^{j+4} + 3^{j+2}$?
76. q varies jointly with the square of r and the square root of t . If $q = 36$ when $r = t = 36$, what value of r will cause q to equal 216 when $t = 144$?
77. The vertex and x-intercepts of the parabola with equation $y = 2x^2 - x - 15$ are used as the vertices of a triangle. What is the area of the triangle?
78. What is the product of the three complex cube-roots of -27?
79. What is the least common multiple of 548 and 458?
80. 1 m unit cubes are used to build a block measuring 4 cm by 8 cm by 6 cm. A tiny ant then chews his way in a straight line from one vertex of the block to the furthest vertex. How many cubes does the ant pass through? The ant is so tiny that he does not “pass through” cubes if he is merely passing through where their edges or vertices meet.

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81. The palindromes larger than 12345 are written in a list in ascending order. What is the 111th member of this list?
82. What is the next term of the sequence beginning with 14, 32, 40, 48, 66, 72, 92, 108, 118, and 162?
83. What is the sum of the perfect squares from 100 to 400, inclusive?
84. What is the sum of the twelve smallest positive perfect cubes?
85. What is the sum of the positive two-digit multiples of 6 that are not written using the digit 6?
86. A trusted friend flips six coins and tells you that there were at least two each of heads and tails. What is the probability that there were three of each?
87. Wendy and Xi love to play Flip!, in which they take turns flipping a coin and keeping track of the results. Once the same result has been flipped twice in a row (e.g. two heads in a row or two tails in a row), the player who flipped the second one is crowned the winner. What is the probability that the first player wins?
88. Because you're so nice, Mathy Noll lets you select one of four identical gift-wrapped boxes from his Table of Gifts. He says he remembers that one of the boxes contains one million dollars, and that the others contain leftovers from various meals he's eaten, but **he's not sure which is which**. You figure there's not much to lose, so you touch a box with the intent of opening it. Suddenly Mathy says "I have to know!" and unwraps one of the other boxes ridiculously quickly, revealing some moldy pizza in a tupperware. "I'm sorry," he says, "you can switch your choice if you want to." If you take advantage of his offer and select one of the other two boxes (not the one you picked initially, nor the one he opened), what is the probability it contains the million dollars?
89. What is the shortest distance from the point $(1,1,1)$ to the plane $2x - y - 2z = 11$?
90. In a seven-element data set of integer test scores from 0 to 100 inclusive, the mean is 58, the median is 68, and the range is 42. What is the largest possible value of the unique mode?
91. In the data set $\{8,93,452,90,3,456,8,d,f\}$, d and f are integers, and the mean is greater than the median, which is greater than the unique mode. What is the smallest possible sum of d and f ?
92. Set J is the set of one-digit counting numbers, and Set K is the set of all counting numbers less than 20. How many sets are subsets of K, supersets of J, contain at least two prime numbers, and at most six composite numbers?

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93. When Aryc, Brynn, Candyce, and Dylan got together for game night recently, they brought their children (Myron, Nancy, Orly, and Penny), their pets (Elly, Freddy, Gryphon, and Hungry), some dessert to share (Ice Cream, Juicers, Kit Kats, and Lemon Bars). The following statements are all true:

The Lemon Bars and Penny came together, as did the Ice Cream and Freddy.
Gryphon, Myron, and the Lemon Bars all came separately, but Elly & Nancy came together.
None of Hungry, Penny, or the Juicers came with Dylan.
Aryc brought the Kit Kats, Brynn did not bring Elly, and Candyce brought Orly.

What three things did Dylan bring?

94. All Knights speak only truths, all Knaves speak only lies, and all of the people below are one or the other.

Person Z: Y is not a knave.
Person Y: Neither Z nor W is a knave.
Person X: At least one of Y or Z is a knave.
Person W: Both of X and V are knights.
Person V: I am not a knave.

List the letters of all of the people above who **must** be knaves.

95. A triangle has an angle of 120° between sides measuring 3 m and 4 m. What is the length, in meters, of the third side?
96. A triangle has sides measuring $\sqrt{10}$ m, $\sqrt{17}$ m, and $\sqrt{29}$ m. What is its area, in meters?
97. What is the sum of the values of n between 0 and 2π that satisfy $\sin n + 2 \cos^2 n = 1$?
98. If $q(r) = 3^{2r}$, evaluate $\frac{dq}{dr}$ when $r = -\frac{1}{2}$.
99. Petra is using a 20 ft. ladder to spray Formula 409 on a window 19 ft. above the ground. Suddenly the base of the ladder begins to slip away from the wall! Strangely, the ladder's base slides smoothly at a speed of 2 ft. per second while the top of the ladder slides smoothly down the wall of the house. How fast, in feet per second, is the top of the ladder sliding down when the base of the ladder is 16 ft. from the house?
100. What is the average value of the function $k(m) = 3m^4$ between $m = 2$ and $m = 3$?