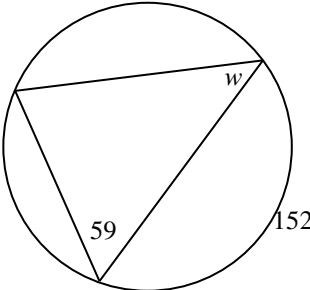


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1. Simplify: $\sqrt{5200}$
2. Evaluate: $\frac{5}{6} - \frac{3}{8}$
3. Express .00077652 in scientific notation rounded to three significant figures.
4. Evaluate: $\frac{11! \times 3!}{5! \times 7!}$
5. Evaluate: $\binom{7}{4}$
6. Express in simplest radical form: $\sqrt[3]{875}$
7. Arrange the letters in **decreasing** numerical order:
 $Z = 13 \times 17$ $Y = 987 - 678$ $X = 5432 \div 19$ $W = 123 + 421$
8. What ordered quadruple, of the form (r, s, t, u) , satisfies the equations $2r + 2s + 2t + 3u = 10$, $2s + 3t + 2u + 2r = 11$, $2t + 2u + 3r + 2s = 6$, and $2u + 2r + 3s + 2t = 9$?
9. What value(s) of g satisfy $5(78g - 90) + 6(12 - 34g) = 16944$?
10. Amy gives half of her marbles to Billy, then buys 47 marbles, then gives a third of her marbles to Celia, after which she gives 74 marbles to Davie, ending with 174 marbles. How many marbles did Amy start with?
11. What value(s) of c satisfy $7c + 5 = 3c - 2$?
12. What value(s) of b satisfy $2b^2 - 5b - 12 = 0$?
13. If you travel at a constant speed of 79 meters per second for one second less than a minute, how many meters do you travel?
14. You bike uphill to school at an average speed of 4 meters per second, but bike home at a downhill speed of 9 meters per second. What was your average speed, in meters per second, for the round-trip to and from school?
15. Express the equation of the line through the points $(-3, 5)$ and $(1, -1)$ in slope-intercept form ($y = mx + b$).
16. What are the coordinates, in the form (x, y) , of the midpoint of the line segment connecting the points $(-9, -8)$ and $(2, -4)$?
17. What are the coordinates, in the form (x, y) , of the point of intersection of the lines $y = 2x + 3$ and $3x - 2y = 1$?
18. The point $(-3, 7)$ is rotated 90° counter-clockwise about the point $(5, 2)$ to Point D. What are the coordinates, in the form (x, y) , of Point D?

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19. What are the coordinates, in the form (x, y) , of the rightmost x-intercept of the parabola $y = -3x^2 + 12x - 4$?
20. Mr. Jacobson asks his students to find the roots of a parabola of the form $r^2 + Br + C = 0$ on the screen. Megan miscopied the value of B and got roots of 6 and -4. Greg miscopied the value of C and got roots of -6 and 1. What are the correct roots of Mr. Jacobson's parabola?
21. In Tunnel Hill, elves tend giant ants as livestock. If there are a total of 100 legs and 20 heads, how many elves are there?
22. If $g(h) = \frac{(h+4)^2}{h-3}$, evaluate $g(10)$.
23. In the figure to the right with a circle circumscribed around a triangle, an arc measure and two of the triangles angles are given in degrees. What is the value of w ?
- 
24. How many regular polygons can tessellate a plane all by themselves?
25. A rectangle measures 8 m by 12 m. What is the area, in square meters, of all of the points that lie within one meter of at least one point on the rectangle?
26. How many diagonals can be drawn in a convex 15-gon?
27. A right triangle has a hypotenuse measuring 9 m and a leg measuring 5 m. What is the length, in meters, of the other leg?
28. A triangle with angles measuring 30° , 60° , and 90° has a longest side measuring 18 m. What is the area, in square meters, of the triangle?
29. A triangle with angles measuring 45° , 45° , and 90° has a longest side measuring 18 m. What is the length, in meters, of the shortest side of the triangle?
30. When the three medians of a triangle are drawn, they will always meet at a single point. What is a name for this point?
31. Two sides of a triangle measure 41 m and 56 m. When the third side of the triangle is measured in meters, what is the smallest possible integer value of that measurement?
32. What is the area, in square meters, of a hexagon with a perimeter of 48 m?
33. What is the circumference, in meters, of a circle with an area of 49π square meters?
34. What is the volume, in cubic meters, of a right circular cone with a base radius of 12 m and a height of 5 m?
35. The supplement of the complement of an angle measures 119° . What was the measure, in degrees, of the original angle?

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36. Regular 24-gon Z has vertices labeled A-X in clockwise order. Evaluate $m\angle ADV$, in degrees.

37. What value(s) of q satisfy $4^{2q} = 48 \times 4^q - 512$?

38. Use the table to the right to evaluate

$$f\left(g\left(h^{-1}\left(f\left(g^{-1}(1)\right)\right)\right)\right).$$

	$z = 1$	$z = 2$	$z = 3$	$z = 4$	$z = 5$	$z = 6$
$f(z)$	1	5	2	6	4	3
$g(z)$	4	2	1	5	3	6
$h(z)$	6	4	3	2	5	1

39. The function $j(k) = \frac{2k^2}{k^2+1}$ has a domain and a range that are subsets of the real numbers. Express the range in interval notation.

40. What is the product of the reciprocals of the roots of $3m^3 - 2m^2 + m - 4 = 0$?

41. What are the coordinates, in the form (x, y) , of the center of the conic with equation $9x^2 - 2y^2 - 36x - 24y = 100$?

42. How many positive integers are factors of 540?

43. What is the product of the smallest and largest three-digit palindromes?

44. List which of the following numbers are divisible by 12:

53142 97568 35962 14796 98632 18514 82608

45. Event R has a probability of $\frac{1}{4}$ and Event S has a probability of $\frac{2}{3}$. If these events are mutually exclusive, what is the probability of $R \cap S$?

46. In how many distinguishable ways can the letters in the word "TALLAHASSEE" be arranged?

47. What is the sum of the perfect squares less than 1000?

48. What is the 1234th term of an arithmetic sequence with a first term of 56 and a common difference of 78?

49. What is the smallest possible sum of an infinite geometric sequence with a third term of 18 and a fifth term of 2?

50. Evaluate: $\langle 1, 3, -5 \rangle \cdot \langle 4, -2, -1 \rangle$

51. In a five-element data set of integer test scores from 0 to 100 inclusive, the mean is 70 and the median is 60. What is the smallest possible value of the range?

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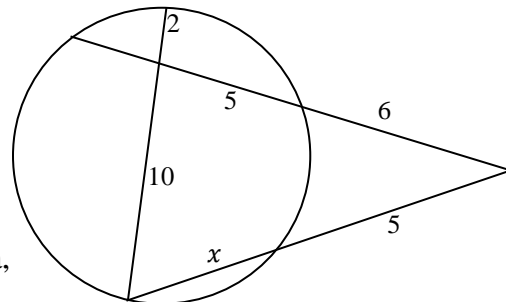
52. What is the product of the median and range of the data set $\{7, 2, 9, -3, 5\}$?
53. Set B is the set of positive even numbers less than 20, and Set L is the set of positive multiples of 4 less than 10. How many subsets of set B are supersets of Set L?
54. What is the measure of the angles at the corners of the gigantic rectangular geologic feature recently discovered on the moon?
55. Convert $\frac{\pi}{7}$ into degrees, minutes, and seconds to the nearest second in the form $d^{\circ}m's''$.
56. If $\sin k = \frac{5}{13}$, what is the largest possible value of $\sin \frac{k}{2}$?
57. Express the cylindrical coordinates $(8, \frac{5\pi}{3}, 2)$ in rectangular form (x, y, z) .
58. What is the area of the region bounded by $2 < x < 4$ and $0 < y < x^3$?
59. Evaluate **as a mixed number**: $5 \frac{11}{135} \div 1 \frac{37}{75}$
60. Evaluate: $65^3 - 35^3$
61. Simplify by multiplying and combining like terms: $(v - 1)v(2v + 8)(3 - v)$
62. Express the solution to the system of equations $u + 2t + 3s = -6$, $2u - 4t + s = 9$, and $3u + 2t - 2s = 1$ as an ordered triple in the form (u, t, s) .
63. Consider the graphs of Parabola H ($y - k = a(x - h)^2$) and Parabola J ($y - k = a(x - j)^2$), where a, h, j , and k are constants and $h > j$. List all of the following phrases that would be accurate in the sentence "Parabola H is _____ Parabola J."
- | | | | |
|-------|-----------------|---------------|--------------|
| above | to the left of | narrower than | taller than |
| below | to the right of | wider than | shorter than |
64. All the families in the cul-de-sac contribute the same amount of money towards the purchase of a riding lawn mower that anyone can use to mow their lawn. If there had been one fewer family, each family would have had to pay \$274 more than they actually did. If there had been three more families, each family would have paid \$685 less than they actually did. How many dollars did the riding mower cost?
65. I've got 37 coins in my pocket worth a total of \$6.00. If I have at least one penny, nickel, dime, and quarter, and no other types of coins, what is the largest number of nickels I could have?
66. A rectangle with an aspect ratio of 2 is inscribed in a circle with a radius of 3. What is the area, in square meters, of the rectangle?

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67. When the measures of seven of the angles of a convex decagon are added together, the result is 987° . If all angles are integers when measured in degrees, what is the smallest possible number of degrees in the measure of one of the three other angles?

68. What is the volume, in cubic meters, of a regular octahedron with edges measuring 8 m?

69. In the figure to the right, several line segments intersect a circle and one another, and all segment lengths are given in meters. What is the value of x ?



70. Two chords in a circle are perpendicular, and divide each other into lengths of 2 m, 3 m, 4 m, and t m (in no particular order). What is the largest possible area, in square meters, of the circle?

71. Two circles with respective radii of 8 m and 16 m have their centers 30 m apart. What is the length, in meters, of one of their common internal tangents?

72. What is the largest real value of r for which there is a real value of s satisfying $3r^2 + 2s^2 + rs + 3r + 2s = 6$?

73. At how many points do at least two of the graphs of $x^2 + \frac{y^2}{4} = 100$, $(x - 1)^2 - \frac{(y+2)^2}{4} = 25$, and $\frac{(x+3)^2}{9} + (y - 4)^2 = 49$ intersect?

74. If $\log_2 3 = n$ and $\log_3 5 = p$, evaluate $\log_{10} 27$.

75. Express the base nine number 248_9 as a base three number.

76. If $q \equiv 3 \pmod{6}$ and $q \equiv 5 \pmod{8}$, what is the smallest possible value of $q > 1000$?

77. Unit squares are used to create a 12 by 18 rectangle, and one of its diagonals is drawn. How many unit squares does this diagonal pass through (merely touching a corner does not count)?

78. A trusted friend rolls two standard dice behind a screen and tells you she didn't get any odd numbers. What is the probability that she rolled a sum of six?

79. If two points are chosen on a meter stick, what is the probability they are within 20 cm of one another?

80. Two people play a game in which they take turns rolling a fair six-sided dice. They take turns until somebody wins by rolling a number equal to or greater than the number just rolled by the other player. According to these rules, the first player cannot win on her first turn. What is the probability that the second player wins the game eventually?

81. Five distinguishable keys, each with a flat side and a bumpy side, are placed on a standard key ring. How many distinguishable arrangements are possible?

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82. As the millionth customer at the casino, you're going to win money by playing a special game! You'll flip a coin eight times; if your first flip is heads, you'll win \$1. If your first two flips are heads, you'll win an additional \$2. If your first three flips are all heads, you'll win an additional \$6. If your first four flips are all heads, you'll win an additional \$24. If your first five flips are all heads, you'll win an additional \$120. If your first six flips are all heads, you'll win an additional \$720. If your first seven flips are all heads, you'll win an additional \$5040. Finally, if you flip either eight heads or eight tails, you'll win \$1,000,000! (For all heads, the \$1,000,000 is in addition to your earlier winnings.) What are your expected winnings, in dollars rounded to the nearest hundredth (cent)?
83. At the same casino, you're allowed to draw a marble from a bag containing two red and three green marbles, but you can't look at it; you just keep it clutched in your hand. After you choose, the Pit Boss roots around in the bag (he looks) and removes two green marbles, which he shows you and then throws into the watching crowd. After this, he gives you a chance to throw your marble into the crowd (still not looking, and the crowd remains deathly silent) and select a new marble from the bag. You could also choose to keep the marble in your hand (without looking at it). If you end up with a red marble, you Win Big and get Lots of Money, otherwise you Lose and get Nothing. If you choose to throw your first marble away and select a second marble, what is the probability you end up with a red marble?
84. Evaluate: $\sum_{z=2}^{\infty} \frac{1}{z^2 - z}$
85. What is the missing term of the sequence $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{24}{5}, 20, \frac{720}{7}, \underline{\hspace{1cm}}, 4480, 36288, \dots$
86. What is the ninth term of a recursive sequence with first term $t_1 = 1$ and nth term $t_n = 2t_{n-1} + 3$?
87. What is the sum of the positive four-digit even numbers?
88. The matrix $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ has an infinite number of eigenvectors in the form $\begin{bmatrix} x \\ y \end{bmatrix}$. List the two which are linearly independent of one another, have a positive x-component, and have components which are relatively prime integers.
89. In the data set of integers $\{8, u, -2, v, 4, w, 1, 4, 2\}$, the single mode is greater than the median, which is greater than the mean. What is the largest possible value of $u + v + w$?
90. Set V is the set of people who like Vanilla, and contains 31 elements. Set C is the set of people who like Chocolate, and contains 41 elements. If the set of all people surveyed contains 61 people, what is the largest possible number of elements in the set $V' \cup C'$?
91. In the cryptarithm $STE + MATH = STEM$, each instance of a particular letter represents the same digit (0-9) and no two different letters represent the same digit (e.g. if an A is a 1, then all A's are 1's and B's are not 1's). What is the largest possible value of the five-digit number STEAM?

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92. You invite eight friends (A-H) over to watch a movie, but they all decline. You press them on the issue, and get the following responses in order, with each of them having heard all of the earlier responses:

A: I'm going on a date with D tonight.
B: That's a lie!
C: G & I are going on a date tonight.
D: I'm going on a date with E tonight.
E: I'm planning to wash my car tonight.
F: No one has told the truth yet!
G: I'm going on a date with C tonight.
H: At least three people have told the truth so far.

"I" refers to the speaker, rather than to another friend. Each friend is only doing one thing tonight, each date involves exactly two people, and washing a car is NOT a date. Of the eight statements, the smallest possible number are lies. List the letters of the people who **must** have told you the truth.

93. When ten friends sit in a row at the movies, Annie sits within two seats of Billy (at most one person between them), Cindy does NOT sit within three seats of Darian, Elaine sits next to Francis, Ginny does NOT sit next to Harry, and Indiana sits within two seats of Justine. In addition, Annie sits somewhere to the left of Cindy, Billy sits somewhere to the right of Elaine, Darian sits immediately to the left of Francis, Ginny sits immediately to the right of Justine, and Harry and Indiana do NOT sit within two seats of one another. Finally, Cindy sits on an end, Harry is one of the two centermost friends, Billy sits somewhere to the left of Ginny, and Annie sits next to Justine. Write the first letters of each friend in the order they sit from left to right. E.g. AJBICHGDEF.

94. Evaluate:
$$\frac{1}{2 + \frac{3}{4 + \frac{1}{2 + \frac{3}{4 + \dots}}}}$$

95. An ant is on an edge of a cube two centimeters from a vertex, and wishes to reach a point on an edge two centimeters from the opposite vertex. What is the minimum number of centimeters the ant must walk if the cube's edges are nine centimeters long?

96. In how many points do the graphs of $y = \log_{10} x$ and $y = \sin(\pi x)$ intersect?

97. Simplify into a product of two of the six basic trigonometric functions:

$$\sin u \cos^2 u \tan u + \cos(u) \left(\frac{\cos u}{\sec u} + \cot^2 u \right)$$

98. An athlete throws a 10 pound shot put at an angle 30° above the horizontal and releases it at a speed of 192 feet per second. If the shot put lands on a shelf at the same height as the shot put was released, how far is the shelf from the athlete? Use the standard whole number for the acceleration due to gravity in $\frac{ft}{sec^2}$.

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98. An athlete throws a 10 pound shot put at an angle 30° above the horizontal and releases it at a speed of 192 feet per second. If the shot put lands on a shelf at the same height as the shot put was released, how far is the shelf from the athlete? Use the standard whole number for the acceleration due to gravity in $\frac{ft}{sec^2}$.
99. Express $\lim_{n \rightarrow 2} \frac{1 - \frac{1}{2^n}}{2 - n}$ in terms of a function $f(x)$ and its derivative at a point (e.g. $f(x) = x^2$, $f'(-3)$).
100. For the function $g(x) = x^2 + 4$ defined for $-1 < x < 3$, what is the value of c for which $g'(c)$ satisfies the Mean Value Theorem?