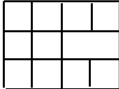


2012 Team Scramble  
Thursday, November 8th, 2012

This test consists of 100 problems to be solved in 30 minutes by a large team. All answers must be exact, complete, and in simplest form. To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem. A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate:  $1234 + 567 + 89012 + 34$
2. What number is eight more than twice the product of nine and the sum of ten and eleven?
3. What percent of 180 gives 27?
4. Evaluate:  $-3 - (-2)^{-1}(-9)$
5. Evaluate:  $\frac{4! \times 7!}{6! \times 5!}$
6. Express in simplest radical form:  $\sqrt{\frac{12 \times 18 \times 20 \times 32}{15 \times 6}}$
7. Evaluate:  $41^2 - 39^2$
8. Arrange the variables A-E in increasing order (e.g. your answer might be ABCDE).  
 $A = \binom{100}{97}$        $B = \sqrt{200,000}$        $C = 2.3^{45}$        $D = 98 \times 76$        $E = 8!$
9. When my secret number is decreased by thirteen, this result is tripled, and that result is increased by thirty, the final result is 2325. What is my secret number?
10. What value(s) of  $f$  satisfy  $97f + 89 = 83 - 79f$ ?
11. What value(s) of  $g$  satisfy  $6(5g - 4) + 3 = 2 - 9(8 + 7g)$ ?
12. What value(s) of  $h$  satisfy  $12h^2 - 88h - 15 = 0$ ?
13. The sum of two numbers is 987 and their difference is 567. What is the smaller of the two numbers?
14. Express the equation of the line through the point (3,5) and parallel to the line  $3x + y = 7$  in slope-intercept ( $y = mx + b$ ) form.
15. What is the shortest distance from the point  $(-2,9)$  to the line  $y = 7x + 4$ ?
16. What are the coordinates, in the form  $(x, y)$ , of the vertex of the parabola  $y = 2x^2 - 6x + 7$ ?
17. What is the smallest possible **positive** difference between a positive four-digit integer and the positive four-digit integer formed by reversing its digits?

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18. At the Longneck Ranch, a corral contains Emus & Llamas. If there are 44 heads and 144 feet, how many Emus are in the corral?
19. In a set of three numbers, the sums of the two-element subsets are 14, 31, and 9. What is the value of the smallest number?
20. Two circles have radii of eight meters and ten meters, and their centers are twelve meters apart. What is the length, in meters, of one of their common external tangents?
21. What is the perimeter, in meters, of an equilateral triangle with an area of  $96\sqrt{3} \text{ m}^2$ ?
22. What is the name of a triangle where each angle measures less than  $90^\circ$ ?
23. What is the perimeter, in meters, of a convex polygon with sides measuring 9 m each where the number of sides of the polygon equals its number of diagonals?
24. What is the area, in square meters, of an isosceles right triangle inscribed in a circle with a radius of 8 m?
25. What is the general name for a polygon with four sides?
26. A rhombus with a perimeter of 12 m and an area of  $8 \text{ m}^2$  is similar to another rhombus with a perimeter of 16 m. What is the area, in square meters, of the larger rhombus?
27. How many edges does a regular dodecahedron have?
28. In a triangle with sides measuring 7 m, 9 m, and 12 m, what is the length, in meters, of the altitude to the shortest side?
29. A cow is tied to an external corner of a rectangular barn with sides measuring 20 m by 40 m. If the length of the cow's rope is 50 m, what is the area, in square meters, of the region in which the cow can graze?
30. How many squares of any size are there in the array of unit squares shown with one missing segment?
31. What is the largest number of regions into which three pairs of perpendicular lines can divide a plane?
32. A regular polygon has vertices lettered sequentially around its circumference: A, B, C, ... If  $\overline{FN}$  would bisect the figure, what line segment connecting vertex J to another vertex would also bisect the figure?
33. What is the length of the latus rectum of the parabola with equation  $y = 3x^2 - 4x$ ?
34. If \$100 is invested at 10 percent annual interest compounded continuously, how much money, in dollars rounded to the nearest dollar, will be in the account after five years?

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35. Evaluate:  $\log_{81} 27\sqrt{3}$
36. If  $a(b) = 2 \times 3^{4b} - 5$ , evaluate  $a^{-1}\left(-\frac{43}{9}\right)$ .
37. If  $c(d) = 9d + 8$  and  $f(g) = g^2 - g$ , evaluate  $f(c(-2))$ .
38.  $h$  varies jointly as the square of  $k$  and inverse of  $j$ . If  $h = 36$  when  $j = 24$  and  $k = 12$ , what will  $h$  be when  $j = 6$  and  $k = 6$ ?
39. Smithium has a half-life of 20 minutes. How many **grams** of a 100 kg sample of Smithium will remain after two hours, **as a decimal**?
40. What is the product of the roots of  $m^5 - 2m^4 + 3m^2 = 4$ ?
41. How many multiples of 18 are factors of 1782?
42. Evaluate:  $\prod_{f=2}^{34} \left(1 - \frac{1}{f}\right)^2$
43. What is the total number of rectangles of any size on a checkerboard? Hint: there are eight rows and eight columns on a checkerboard.
44. When six fair coins are flipped, what is the probability that there are more heads than tails?
45. What is the equation of the plane through the point  $(1, -2, -3)$  and perpendicular to the line through the points  $(2, 3, -4)$  and  $(-5, 1, -3)$ ? Please write your answer in the form  $Ax + By + Cz = D$ , where  $A$  is positive and  $A, B, C$ , and  $D$  are integers that do not collectively share a common factor.
46. Set N is the set of positive two-digit integers the sum of whose digits is nine. Set P is the set of positive two-digit integers that are multiples of six. How many subsets of Set N are also subsets of Set P?
47. Set Q is the set of prime numbers between 0 and 10, and Set R is the set of prime numbers between 30 and 40. How many distinguishable functions have Set Q as their domain and some subset of Set R as their range?
48. Willa is taller than both Vince and Umberto, Tom is shorter than Sylvia, and Vince is taller than Sylvia. If they line up from shortest to tallest, how many orders might be possible?
49. An ant is at the midpoint of an edge of a regular octahedron and wishes to reach the midpoint of the opposite edge. If an edge of the octahedron measures 12 m, what is the shortest distance, in meters, that the ant can walk?
50. What is the area, in square meters, of a triangle with sides measuring 8 m, 13 m, and 15 m?
51. What is the area, in square meters, of a triangle with two sides measuring 6 m and 8 m and the angle between them measuring  $15^\circ$ ?

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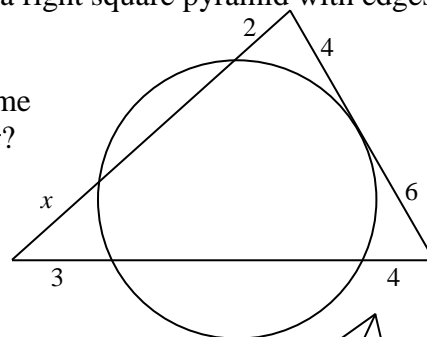
52. Evaluate in radians:  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
53. If  $b(c) = \frac{6c^2+5c-4}{2c-3}$ , evaluate  $b'(-2)$ .
54. If  $y = \begin{cases} 3x + 2 & x < 1 \\ x^2 + Ax + B & x \geq 1 \end{cases}$ , what ordered pair  $(A, B)$  will make the function differentiable for all values of  $x$ ?
55. What is the equation, in the slope-intercept form, of the line tangent to the graph of  $y = x^3 - x + 4$  at the point  $(-1, 4)$ ?
56. Evaluate:  $1452048 \div 312$
57. How many teaspoons are in seven gallons?
58. Round to the nearest hundred:  $\frac{12345+6789}{987-98}$
59. Simplify by rationalizing the denominator:  $\frac{162}{9+3\sqrt{7}}$
60. What ordered pair  $(j, k)$  satisfies the system of equations  $11j - 7k = 5$  and  $3j + 2k = 17$ ?
61. If Eric can build a house in 24 days and Tom can build one in 36 days, how many days would it take them to build a house if they worked together?
62. If nine chickens can lay 24 eggs in three days, how many days would it take seventeen chickens to lay 272 eggs?
63. If Li travels at 40 kmph for half the distance she intends to travel, but wants to average 50 kmph for the entire trip, what speed (in kmph) should she average for the second half of her trip?
64. What is the distance between the  $x$ - and  $y$ -intercepts of the line  $2x + 3y = 45$ ?
65. A cowboy is at the point  $(5, -3)$  and wishes to ride to the river (represented by the line  $y = 19 - x$ ) before heading to town at  $(-2, 11)$ . What is the shortest distance he can ride?
66. When Mr. Brown writes a quadratic of the form  $m^2 + Nm + P = 0$  on the board, Quynh writes the wrong value of  $N$  and gets roots of 4 and -7, while Rowan writes the wrong value of  $P$  and gets roots of -2 and -3. What are the roots of Mr. Brown's original equation?
67. A rectangular photograph with a perimeter of 48 cm and an area of  $128 \text{ cm}^2$  is surrounded by a rectangular frame with each of its edges exactly 3 cm from an edge of the photograph. What is the area, in square centimeters, of just the frame?

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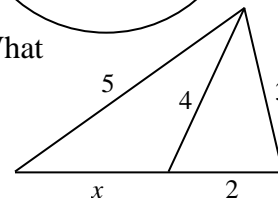
68. A group of coworkers pooled their money to buy BiggiBillions lottery tickets, and ended up winning! Of course they split the winnings equally. If there had been one more coworker involved each of them would have received thirty million dollars less, while if there had been two fewer coworkers involved each of them would have received eighty million dollars more. What was the size of the jackpot, in billions of dollars **expressed as a decimal**?
69. If  $6stu = -20$ ,  $s + 2t + 3u = 5$ , and  $2st + 6tu + 3su = -4$ , and  $s > t > u$ , evaluate  $s + t + u$ .
70. Seven years ago, Valerie was eight years younger than Willa will be when Xavier is 11. In six years, Willa will be three times as old as Xavier and five years older than Valerie is currently. What is the sum of their current ages?

71. What is the maximum possible volume, in cubic meters, of a right square pyramid with edges measuring 3 m and 5 m?

72. In the intersecting circle and triangle shown to the right, some segment lengths are given in meters. What is the value of  $x$ ?



73. In the triangle to the right, all segment lengths are given in meters. What is the value of  $x$ ?



74. Three congruent beach balls with radii of 30 cm are all touching one another while sitting on the floor. Resting on top of the three is a smaller ball with a radius of 15 cm. What is the vertical distance, in centimeters, from the top of this ball to the floor?
75. What are the coordinates, in the form  $(x, y)$ , of the rightmost focus of the hyperbola  $9x^2 - 16y^2 + 54x + 32y = 79$ ?
76. What is the largest real value of  $n$  for which there is a real value of  $p$  satisfying  $3n^2 + 4p^2 - 5np + n = 6$ ?
77. What is the area of a triangle with vertices at the focus and x-intercepts of the parabola  $y = 3x^2 - 10x - 8$ ?
78. Evaluate as a base 12 number, where the digit A represents 10 and the digit B represents 11:  
 $B9A_{12} \times AB8_{12}$
79. How many positive integers are factors of 4998?

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80. Arithmetic sequence C has a first term of 2109 and a common difference of 1350, and arithmetic sequence D has a first term of 3456 and a common difference of 8262. What is the smallest possible positive difference between a term of sequence C and a term of sequence D?
81. What is the 10,000th smallest positive palindromic integer?
82. What is the sum of the multiples of seven between 699 and 7778?
83. Sequence A is an arithmetic sequence with first term 114 and common difference 4 and Sequence B is a geometric sequence with first term 3072 and common ratio  $\frac{3}{2}$ . What is the sum of the terms of Sequence A that are also in Sequence B?
84. When Giana is dealt three cards from a standard 52-card deck, she tells you that she does not have three of the same suit. What is the probability that she has three of the same rank?
85. In how many ways can five nickels and ten dimes be arranged in a line if no two nickels may be adjacent to one another?
86. Helga's High School has 314 students. 37 students from the Math Club are also in Glee Club, 23 Glee Club students are also in Key Club, and 19 Key club members are also in Wii Club. No student is in exactly three of the clubs, and four times as many students are in all four of the clubs as are in exactly one of the clubs. If there are more students in Math Club than Glee Club, more in Glee Club than Key Club, and more in Key Club than Wii Club, what is the minimum number of students in Math Club if every student is in at least one of these four clubs?
87. In an unusual game, two players take turns rolling a die with colored faces. The numbers 1, 2, 3, and 4 are colored red and the numbers 5 and 6 are colored blue. The first player wins if he rolls a 1 on his first roll. Otherwise, a player wins if they roll a number with the same color as the number just rolled by the other player. What is the probability that the first player wins?
88. Ian's boss Jenny has an unusual way of paying him. In theory, Ian's salary is \$500 a week paid every week, but instead each Friday Jenny flips a coin three times. The first flip can either double or halve Ian's paycheck. The last two flips work together: If they're both heads, Jenny deducts \$100 from Ian's paycheck; if they're both tails Jenny deducts \$300 from Ian's paycheck; if they're a head and a tail Jenny adds \$500 to Ian's paycheck. If the final result is negative, James is paid nothing, but doesn't have to pay Jenny for the privilege of working for her. What is the expected value of Ian's paycheck in dollars rounded to the nearest hundredth (cent)?
89. What is the area of the triangle with vertices at the points (1,2,3), (-2,3, -1), and (4, -3,0)?
90. In a set of nine integer test scores from 0 to 100 inclusive, the range is 71, the unique mode is 69, and the mean is 65. What is the smallest possible value of the median?

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91. The data set  $\{5, 10, 11, 14, 18, 20\}$  has three other positive integer elements,  $x$ ,  $y$ , and  $z$ . If the unique mode is less than the mean, which in turn is less than the median, what is the largest possible sum of  $x$ ,  $y$ , and  $z$ ?
92. Set  $L$  is the set of positive three-digit integers whose units digit is smaller than their tens digit, which in turn is smaller than their hundreds digit. Set  $M$  is the set of positive three-digit integers whose tens digit is equal to the sum of their units and hundreds digits. How many elements are in the set  $L' \cap M$ ?
93. The heist of the century has taken place, and all of the criminal masterminds have been rounded up. We know that exactly two people committed the crime, and we're sure that those two are among the suspects on hand. Each suspect was questioned separately and made the two statements below. Our psychologist assures us that in an attempt to throw us off, all suspects will be sure to make one true statement and one false statement. Which two criminals committed this particular crime? "I" refers to the speaker.
- |                    |                               |
|--------------------|-------------------------------|
| S: I didn't do it! | W & V did it.                 |
| T: OK, I did it.   | U & S did it.                 |
| U: V did it.       | Neither X nor I did it.       |
| V: X did it.       | T did it but X didn't.        |
| W: T didn't do it. | At least one of S & T did it. |
| X: W & I did it.   | At most one of U & W did it.  |
94. If  $\tan a = \frac{3}{4}$  and  $\frac{\pi}{2} < a < \frac{3\pi}{2}$ , evaluate  $\tan\left(\frac{a}{2}\right)$ .
95. What value(s) of  $z$  between 0 and  $2\pi$  inclusive satisfy  $6 \sin^2 z = 4 - 5 \sin z$ ?
96. Simplify:  $\frac{6a^5 + a^4 - 3a^3 + 8a^2 + 18a - 24}{2a^2 - 3a + 4}$
97. What is the area contained by the graph of the parametric equations  $x = -1 - \sqrt{3}\sin t$  and  $y = 1 + \sqrt{5}\cos t$ ?
98. What is the area of the region bounded by the  $x$ -axis and the graph of  $y = 3x^2 + 5x - 2$ ?
99. If  $d(f) = (f + 2)^{f-3}$ , what is  $d'(1)$ ?
100. Use the Trapezoid Rule with  $\Delta x = 1$  to approximate  $\int_2^5 (x^2 + 4)dx$ .