# 2018 Fall Startup Event
Thursday, September 27th, 2018

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate: $553896 - 222716$

2. What is the remainder when 234 is divided by 15?

3. Evaluate: $4 - (-9) - 7(-2 - (-5))(7 - 9) - 4$

4. How many complete months are in 36 years?

5. Express 94.4398637 in scientific notation rounded to five significant figures?

6. Evaluate: $_9C_3$

7. Express in simplest radical form: $\sqrt[5]{96}$

8. Simplify by rationalizing the denominator: $\frac{21}{3-\sqrt{2}}$

9. Evaluate: $82^2 - 78^2$

10. Arrange the letters below in order of ascending value (e.g. BCDA): $A = \frac{2}{6}$, $B = 0.4$, $C = \frac{1}{2}$, $D = 0.71$

11. Express as a simplified fraction: $.\overline{63}$

12. How many dollars are 2 quarters, 3 dimes, 9 nickels, and 4 pennies worth? For example, your answer might be 7.38.

13. If today is Thursday, what day of the week was it 567 days ago?

14. When the secret number is reduced by 35 and this result is divided by 9, the final result is 29. What is the secret number?

15. What value of c satisfies $3c + 54 = 9c - 42$?

16. Simplify by combining like terms: $2f + f^2 - 6 + 9f + 9f^2 + 8 - 4f^2$

17. What value(s) of g satisfy $9g^2 = 9g + 270$?

18. What is the solution, in the form (k, m), of the system of equations $k + 4m = -31$ and $k + 2m = -17$?

19. Zilla could paint the house in 8 hours, and Kong could do so in 12 hours. How many **minutes** would it take them to paint the house if they work together?

20. If 16 chickens can lay 600 eggs in 2 days, how many days would it take for 8 chickens to lay 4200 eggs?

21. When 8 liters of a 60% acid solution is mixed with 2 liters of a 70% acid solution, what percentage of the resulting solution is acid?

22. In a game of Hide & Seek, Pete and Tink see one another when they are 72 feet apart, and the chase is on! If Pete chases Tink at a speed of 6 feet per second, and Tink runs away at a speed of 2 feet per second, how many seconds will it take Pete to catch Tink?

23. Two numbers have a sum of 97 and differ by 39. What is the smaller of the two numbers?

24. What is the equation, in slope-intercept $(y = mx + b)$ form, of the line through the points $(-5, -7)$ and $(9, 35)$?

25. What are the coordinates, in the form (x, y), of the midpoint of the line segment connecting $(-9, 6)$ and $(2, -3)$?

26. Ignoring scaling, which of the following equations might describe the line shown to the right?
    A. $14x - 8y = 6$ B. $9x + 8y = 7$
    C. $8x - 7y = -6$ D. $2x + 4y = -3$
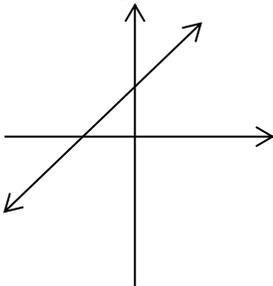
27. What is the equation of the axis of symmetry of the parabola with equation $y = 5x^2 + 60x - 62$?

28. What are the coordinates, in the form (x, y), of the vertex of the parabola with equation $y = 7x^2 + 126x - 49$?

29. When Ms. Tary puts a quadratic of the form $x^2 + Bx + C = 0$ on the board, Wily miscopies the value of B, getting roots of 2 and -9, and JT miscopies the value of C, getting roots of 5 and -8. What is the greater root of the original quadratic?

30. A ranch has only wranglers (humans) and horses. If an automated camera detects 28 heads and 108 legs, how many wranglers are on the ranch?

31. Tom is 7 times as old as Katie. 2 years ago, Katie was 7 years old. How old was Tom then?

32. If $g \blacksquare h = 2gh - h^2$, evaluate $4 \blacksquare 3$.

33. A right triangle has a hypotenuse measuring 39 ft and one leg measuring 36 ft. What is the length, in feet, of the other leg?

34. What is the area, in square meters, of a right triangle with legs measuring 5 m and 14 m?

35. What is the length, in feet, of a leg of a right triangle with one angle measuring 45 degrees and a hypotenuse measuring 8 feet?

36. The figure to the right shows a circle with two intersecting chords, with each line segment's length given in meters. What is the value of w?
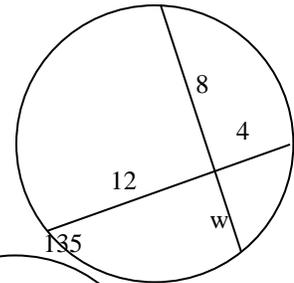
37. How many vertices does a regular dodecahedron have?

38. The figure to the right shows a circle with two intersecting secants with several angle and arc measures given in degrees. What is the value of v?
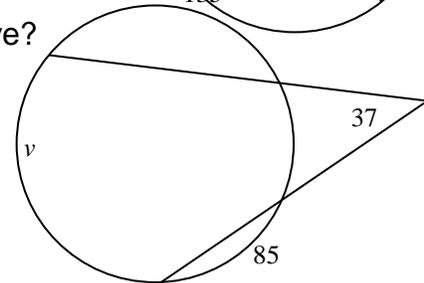
39. What is the circumference, in meters, of a circle with an area of 2π m²?

40. What is the measure, in degrees, of an interior angle of a regular polygon with 8 sides?

41. What is the most specific name for a triangle with two angles measuring 54° and 49°?

42. What is the most specific name that can be applied to all quadrilaterals with two pairs of opposite angles which are congruent?

43. The figure to the right shows three lines, two of which are parallel, with several of the resulting angles labeled in degrees. What is the sum of c and d?
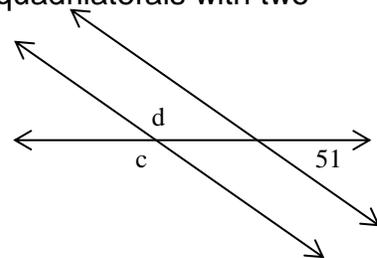
44. A cow is tied to an outside corner of a rectangular barn, all of the doors of which are closed. The barn measures 2 m by 6 m, and the cow's rope is 6 m long. What is the area, in square meters, that the cow can graze?

45. What is the largest floorspace, in square meters, that can be isolated in a corner of a rectangular room by placing a screen that is four feet wide?

46. List which of the following shapes can tesselate an infinite plane by themselves.

47. What is the largest number of regions that 5 lines can divide a plane into?

48. What is the measure, in degrees, of an angle that is complementary to an 81-degree angle?

49. When the vertices of a regular polygon are labeled in clockwise order starting with A, B, C, etc., a line through D and L passes through the center of the polygon. How many sides does the polygon have?

50. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 12:20 AM?

51. A rectangular picture measuring 5 inches by 4 inches has a rectangular frame that is 3 inches wide on each side. What is the area, in square inches, of just the frame?

52. Evaluate: $(1 - i)^3 + (1 + i)^2$

53. What are the coordinates, in the form (x, y), of the center of the locus of points satisfying $2x^2 - y^2 + 8x + 6y = 100$?

54. If $t(s) = 5s - 7$, and $r(q) = 2q + 7$, evaluate $r(t(-2))$.

55. Express the range of $m(k) = 6 - \sqrt{7n + 1}$ in interval notation, given that both the domain and range are subsets of the real numbers.

56. I've invested $10,000 in an account that receives 40% annual interest, compounded quarterly. How much will be in the account at the end of the first year?

57. What is the sum of the roots of $7v^5 - 3v^4 + 3v^3 + 7v^2 - 5v + 3 = 1$?

58. $(2w - 1)^6$ is expanded, like terms are combined, and the terms are arranged in descending degree. What is the middle term, including sign?

59. Simplify: $\frac{6b^4+7b^3-10b^2+4b+8}{b+2}$

60. What is the product of the three complex third roots of $8i$?

61. How many counting numbers between 33 and 50 inclusive are prime?

62. Express the base-9 numeral 270_9 as a base-10 numeral.

63. Express the base-10 number 478_{10} as a base-8 numeral.

64. Express the binary number 101101_2 as a base-4 numeral.

65. What is the prime factorization, in exponential form, of 672?

66. How many positive integers are factors of 756?

67. What is the greatest common factor of 48 and 318?

68. Isaac begins writing the counting numbers in order on the sidewalk, circling every multiple of 80 in purple chalk. Rio follows after him, circling 17 and every 15th number thereafter (32, 47, etc.) with red chalk. What is the smallest positive difference between a red number and a purple number?

69. What is the units digit when 3598^{68} is evaluated?

70. What is the sum of the first 27 terms of the arithmetic (adding or subtracting) sequence whose first three terms are 1, 5, and 9?

71. What is the 5th term of the geometric (multiplying or dividing) sequence whose first three terms are 7, 28, 112?

72. What is the missing term of the quadratic sequence 2, 3, 27, 74, 144, ____, ...?

73. A sequence is defined as $b_n = \frac{1}{n} - \frac{1}{n+1}$. What is the sum of b_5 through b_{18}?

74. What is the missing term of the sequence 1, 6, 15, 12, 29, 24, ____, ...?

75. What is the sum of the 16 smallest positive odd numbers?

76. What is the sum of the 28 smallest positive even numbers?

77. What is the sum of the 19 smallest positive perfect cubes?

78. A bag contains 5 red marbles and 9 orange marbles. When two marbles are drawn without replacement, what is the probability that the first marble drawn is orange and the second marble drawn is red?

79. When three fair coins are flipped, what is the probability that they show exactly 1 tail?

80. In how many ways can 3 purple books, 1 blue book, 2 green books, and 1 yellow book be arranged next to one another on a shelf if books of the same color must all be next to one another?

81. When 84 mathletes were surveyed, 25 said they liked algebra, 34 said they liked geometry, and 19 said they liked both. How many of the students surveyed liked neither algebra nor geometry?

82. A circular dartboard has a **radius** of 7 inches and a bullseye with a **diameter** of 1 inches. What is the probability that a dart that hits a random location on the dartboard will hit the bullseye?

83. Clark and Lois plan to meet at the Daily Planet. Each plans to arrive at a random time between 5 PM and 6 PM, wait up to ten minutes, then leave if the other person isn't there. What is the probability that they actually meet?

84. When a single die is rolled, what is the expected value of the square of the number shown?

85. As I'm about to turn off my light on Halloween, 5 Trick-or-Treaters show up, and I decide to give them everything I have left: 10 identical candy bars. If I don't pay any attention to fairness (all the candy might go to one Trick-or-Treater), in how many ways might I distribute the candy?

86. Evaluate: $< -2, 4, 7 > \cdot < 9, 4, -7 >$

87. What is the shortest distance from the point $(3, -9, 6)$ to the plane $x + 2y - 3z = 4$?

88. What is the range of the data set {35, 15, 25, 41}?

89. What is the mean of the median, mode, and range of the data set {3, 9, 95, 38, 3}?

90. The 12 blue-eyed students in a class averaged 87% on a test, while the 16 brown-eyed students averaged 31%. What was the class average, as a percentage? Note: each student has either blue or brown eyes, but not both.

91. If Set A is {18, 53, 57, 23, 87, 12, 83, 24, 73, 45} and Set B is {84, 56, 12, 65, 38, 94, 80, 95, 27, 98}, what is $A \cap B$?

92. If Set C is {9, 3, 5, 8, 7} and Set D is {9, 1, 7, 8, 2, 3, 5}, how many subsets of Set D are supersets of Set C?

93. In the cryptarithm below, all instances of a particular letter represent the same digit (0-9), and different letters represent different digits. For example, if one A is a 7, all As are 7s and no Bs are 7s. What is the largest possible value of the three-digit number ABC?

$$\begin{array}{r} AB \\ -BC \\ \hline A \end{array}$$

94. In the puzzle to the right, different one-digit numbers (1-9) can be substituted for A, B, C, and D to make all four equations (two across and two down) true. What is the largest possible product of A, B, C, and D?

A	+	B	=	13
-		/		
C	x	D	=	8
=		=		
1		4		

95. If $\sin x = \frac{2}{3}$ and $\frac{\pi}{2} < x < 2\pi$, what is the value of $\cos(x)$?

96. What is the area, in square meters, of a triangle with sides measuring 5 m, 9 m, and 8 m?

97. Express $4 - 4i\sqrt{3}$ in $re^{i\theta}$ form, where $-\pi < \theta \le \pi$.

98. Evaluate: $\displaystyle\lim_{n\to\infty} \frac{2n^2+9n-87}{5n^2-3n+1}$

99. If $f(t) = t^2 \ln t$, evaluate $f''(e)$.

100. If a function satisfies $\frac{dy}{dx} = y + x$ and passes through the point (8,2), estimate the value of y when $x = 7.9$ using a single step of Euler's method.