

2017 Fall Startup Event
Thursday, September 28th, 2017

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction with a whole number denominator unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate: 6×83
2. What is the remainder when 479 is divided by 13?
3. What number is 38 less than twice 97?
4. Evaluate: $\frac{4}{6} - \frac{1}{9}$
5. Evaluate: $-(-5 - (-7)) - 5(-8)$
6. Round 7987.0883 to the nearest ten.
7. In how many ways can you choose three books from a set of nine different books?
8. Evaluate: $65^2 - 55^2$
9. Evaluate: $\frac{10!}{7! \cdot 4!}$
10. Express $\overline{60}$ as a reduced fraction.
11. If today is a Thursday, what day of the week was it 100 days ago?
12. In the number 12345.6789, what digit is in the thousandths place?
13. Simplify: $\frac{4}{\sqrt{2}}$
14. When my secret number is increased by 9 and this result is then multiplied by 8, the final result is 752. What is my secret number?
15. What value(s) of w satisfy $7w - 5 = 135$?
16. What value(s) of v satisfy $6v + 4 = 9v - 8$?
17. Simplify by combining like terms: $7u - 3 - 6u^2 + 1 - 3u + 3 - 2u^2 + 3$

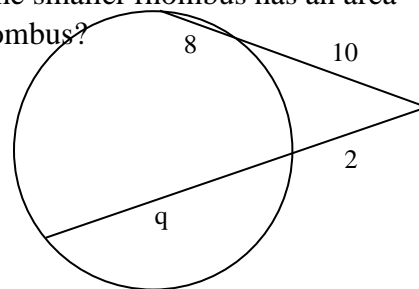
2017 Fall Startup Event
Thursday, September 28th, 2017

18. Olaf & Sven see one another when they are 280 meters apart and immediately start running towards one another. If Sven runs at 4 meters per second and Olaf runs at 3 meters per second, how many seconds will it take them to reach each other?
19. In which quadrant does the point $(-7, -9)$ lie?
20. What is the equation, in slope-intercept form ($y = mx + b$), of the line through the points $(-3, 2)$ and $(-5, 6)$?
21. What is the slope of the line perpendicular to the line $4x + 2y = 8$?
22. What is the distance between the points $(-3, 5)$ and $(3, -5)$?
23. The point $(-1, 1)$ is reflected across the line $x = -7$ to point K. What are the coordinates, in the form (x, y) , of point K?
24. What is the shortest distance from the point $(0, 1)$ to the line $9x - y = -6$?
25. What is the equation of the axis of symmetry of the parabola $x = 8y^2 - 7y - 5$?
26. Katie glues a rectangular picture measuring 9 cm by 26 cm to a rectangular piece of paper so that there is 8 cm of paper showing on each side of the picture. What is the total area, in square centimeters, of the paper that is showing around the edges of the picture?
27. If you can buy S gallons of syrup for D dollars, how many cents would it cost to buy four gallons of syrup?
28. The average age of Nadia, Ori, and James is 8. What will their average age be in four years?
29. I have twelve coins in my pocket worth a total of 52 cents. If the only possible coins are nickels, dimes, and pennies, and there is at least one of each of those, how many dimes are there?
30. What value(s) of h satisfy $\frac{h+7}{h+1} = \frac{h-3}{h-1}$?
31. Simplify by expanding and combining like terms: $(9j - 8)^2$
32. If $k(m) = 6m + 3m^2$, evaluate $k(-3)$.
33. If $n \blacksquare p = np - \frac{n}{p}$, evaluate $8 \blacksquare 4$.
34. A right triangle has legs measuring 7 m and 3 m. What is the length, in meters, of its hypotenuse?

2017 Fall Startup Event
Thursday, September 28th, 2017

35. What is the smallest possible perimeter, in meters, of an isosceles triangle with sides measuring 6 m and 17 m?
36. What is the area, in square meters, of an equilateral triangle with sides measuring 6 m?
37. A right triangle with an angle measuring 45° has a hypotenuse measuring 6 m. What is the length, in meters, of a leg of the right triangle?
38. What is the name of the point where the three medians of a triangle meet?
39. What is the name for a polygon with eight sides?
40. Two similar rhombi have perimeters of 60 m and 40 m. If the smaller rhombus has an area of 30 m^2 , what is the area, in square meters, of the larger rhombus?

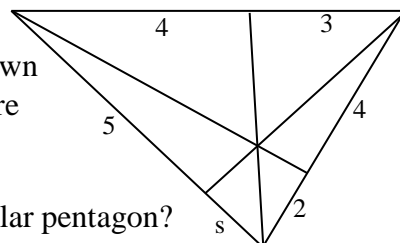
41. How many vertices does a regular dodecahedron have?
42. The figure to the right shows two secant line segments intersecting a circle, with all segment lengths given in meters. What is the value of q ?



43. What is the name for the line segment from a vertex of a triangle perpendicular to the line containing the opposite side of the triangle? Note: sometimes this line segment lies outside the triangle.
44. A right triangle has legs measuring 3 m and 4 m. What is the length, in meters, of the altitude to its hypotenuse?
45. A triangle has two sides measuring 9 m and 16 m. What is the largest integer that could be the length in meters of the third side of the triangle?

46. What is the perimeter, in meters, of a square that is inscribed in a circle with a circumference of 7π ?

47. The figure to the right shows a triangle with three cevians drawn through an interior point, and all perimeter segment lengths are given in meters. What is the value of s ?



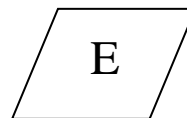
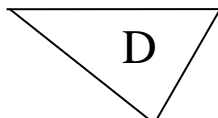
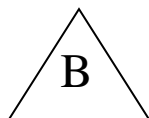
48. What is the measure, in degrees, of an interior angle of a regular pentagon?
49. What is the volume, in cubic meters, of a right circular cone with a height of 3 m and a base radius of 7 m?

2017 Fall Startup Event
Thursday, September 28th, 2017

50. A goat is tethered to an exterior corner of a rectangular shed surrounded by blackberries. The shed measures 2 m by 9 m, and the goat's chain is 6 m long. What is the area, in square meters, of the area the goat can clear of blackberries?
51. Two concentric circles are drawn, and a chord of the larger circle is drawn that happens to be tangent to the smaller circle. If the chord measures 8 m, what is the area, in square meters, of the annular region between the two circles?

52. How many diagonals can be drawn in a convex 15-gon?

53. List the letters of the shapes below that can tessellate (completely cover, without overlapping) a plane by themselves.



54. What is the measure, in degrees, of an angle complementary to one measuring 37° ?
55. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 5:30 AM?
56. A cube of white plastic measures 9 cm on all edges and is painted blue on all faces. When this cube is cut into smaller cubes measuring 3 cm on all edges, how many of those smaller cubes have exactly one blue face?

57. What fraction of the equilateral triangle to the right is shaded? The unshaded region is bordered by two line segments perpendicular to the base, one of which is twice as long as the other.



58. How many real roots does $8n^2 + 9n + 1 = 0$ have?
59. What are the coordinates, in the form (x, y) , of the center of the circle with equation $x^2 + y^2 - 2x + 3y = 21$?
60. What is the area of the ellipse with equation $\frac{(x-2)^2}{15} + \frac{(y+5)^2}{9} = 1$?
61. Evaluate: $\log_2 32$
62. What value(s) of t satisfy $3^{2t} - 18 = 7 \cdot 3^t$?
63. If \$1000 is invested at 10% interest compounded annually, what will its value be after two years? Express your answer in dollars rounded to the nearest hundredth (cent).

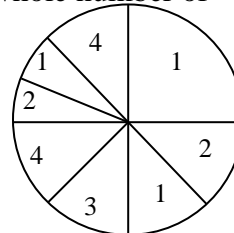
2017 Fall Startup Event
Thursday, September 28th, 2017

64. What is the sum of the roots of $7u^3 + 6u^2 + 8u - 5 = 0$?
65. What is the sum of the prime numbers between 45 and 55?
66. Express the base 10 number 699_{10} as a base 7 number.
67. Express the base three numeral 120210_3 as a base nine numeral.
68. What is the smallest value of $c > 100$ satisfying $c \equiv 3 \pmod{30}$?
69. How many positive integers are factors of 40?
70. What is the sum of the positive integer factors of 48?
71. An 8-by-12 array of unit squares is drawn, as is a line from the upper left corner to the lower right corner of the array. How many of the unit squares does the line pass through?
72. What is the 24th term of an arithmetic sequence with first term 5 and common difference 6?
73. What is the third term of a harmonic sequence with first term 6 and second term 4?
74. What is the missing term of the sequence 3, 2, 6, 10, __, 18, 24, 26, 48, 34, ...?
75. Evaluate: $\sum_{d=1}^8 \left(\frac{1}{d} - \frac{1}{d+1} \right)$
76. What is the sum of the six smallest positive even numbers?
77. What is the sum of the six smallest perfect squares?
78. What is the sum of the eight smallest positive perfect cubes?
79. When two marbles are drawn from a bag containing 8 red and 4 blue marbles, what is the probability that they are different colors?
80. When a single card is drawn from a standard 52-card deck, what is the probability that it is either a heart or a face card (a King, Queen, or Jack)?
81. A trusted friend flips four coins behind a screen and tells you that there are at least two heads showing. What is the probability there are at least three heads showing?
82. When five people run a race, in how many ways can a first-place trophy and a second-place medal be awarded?
83. In how many ways can the letters of "PAPAYA" be arranged?

2017 Fall Startup Event
Thursday, September 28th, 2017

84. In the game of Sparc, a player pays \$5 to roll a single die, and is paid $(n - 2)^2$ dollars if the die showed n on top. What is the expected value, rounded to the nearest whole number of cents, of the person's profit when they play this game once?

85. When playing her favorite board game, what is the probability that Katie gets a 4 when she spins the spinner shown to the right? Assume that central angles are always produced by dividing by two.



86. James wants to wear three matching earrings today, but needs to select them in the dark while his wife is sleeping. If he knows that his earring bowl contains five diamond studs, eleven ruby studs, and nine emerald studs, what is the minimum number of earrings he can take and be certain that he will have three matching earrings?

87. What is the median of the data set $\{48, 83, -79, 96, 818, 382, 30\}$?

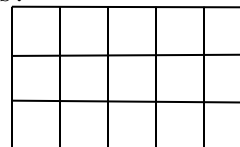
88. What is the mode of the data set $\{91, 69, 97, 32, -25, 36, -57, 97\}$?

89. What is the range of the data set $\{-91, -54, -18, 72, -74, 56, 425, -48, -6\}$?

90. Evaluate: $\langle 1, 4, 5 \rangle \cdot \langle 9, 5, 7 \rangle$

91. How many subsets of $\{2, 3, 5, 8, 13, 21\}$ contain exactly two odd numbers?

92. In the array of unit squares to the right, how many rectangles of any size or shape can be drawn along the gridlines?



93. What is the area, in square meters, of a triangle with sides measuring 5 m, 7 m, and 8 m?

94. What is the period, in radians, of the function $k(m) = 2 \sin(3m) + 4 \cos(5m)$?

95. Express $4e^{-\frac{i\pi}{3}}$ in standard $(a + bi)$ form.

96. What is the most specific name for the shape of the locus of points satisfying $x = \sin t$ and $y = 2 \cos t$?

97. If $n(p) = 3p^4$, evaluate $\frac{dn}{dp}$ when $p = 2$.

98. If $q(r) = (5r - 4)(3 + 2r)$, evaluate $\frac{dq}{dr}$ when $r = -1$.

99. Evaluate $\lim_{s \rightarrow 3} \frac{s^2 - 9}{3s - 9}$

100. Evaluate: $\int_1^2 e^{3w} dw$

2017 Fall Startup Event
Thursday, September 28th, 2017