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1. Evaluate: 6×83

The standard algorithm gives $480 + 18 = 498$.

2. What is the remainder when 479 is divided by 13?

13 goes into 390, leaving 89. 13 goes into 65, then 78, leaving 11.

3. What number is 38 less than twice 97?

$2 \cdot 97 - 38 = 194 - 38 = 156$

4. Evaluate: $\frac{4}{6} - \frac{1}{9}$

$\frac{4}{6} - \frac{1}{9} = \frac{2}{3} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9}$

5. Evaluate: $-\big(-5 - (-7)\big) - 5(-8)$

$-\big(-5 - (-7)\big) - 5(-8) = -2 + 40 = 38$

6. Round 7987.0883 to the nearest ten.

We'll look at the 7, and thus decide to round the 8 up to a 9, giving 7990.

7. In how many ways can you choose three books from a set of nine different books?

$9c3 = \frac{9!}{3!6!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = 3 \cdot 4 \cdot 7 = 84$

8. Evaluate: $65^2 - 55^2$

$65^2 - 55^2 = (65 - 55)(65 + 55) = 10 \cdot 120 = 1200$

9. Evaluate: $\frac{10!}{7! \cdot 4!}$

$\frac{10!}{7! \cdot 4!} = \frac{10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2} = 10 \cdot 3 = 30$

10. Express $.\overline{60}$ as a reduced fraction.

If $b = .\overline{60}$, then $100b = 60.\overline{60}$, so that $99b = 60$, and $b = \frac{60}{99} = \frac{20}{33}$.

11. If today is a Thursday, what day of the week was it 100 days ago?

7 days ago, it was Thursday. 70 days ago, it was Thursday. 28 days before that, it was Thursday. Two days before that was Tuesday.

12. In the number 12345.6789, what digit is in the thousandths place?

The thousandths digit is three digits to the right of the decimal point, and is an 8.

13. Simplify: $\frac{4}{\sqrt{2}}$

$\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

14. When my secret number is increased by 9 and this result is then multiplied by 8, the final result is 752. What is my secret number?

The intermediate result must have been $\frac{752}{8} = 94$, so that the secret number is $94 - 9 = 85$.

15. What value(s) of *w* satisfy $7w - 5 = 135$?

This becomes $7w = 140$, giving $w = 20$.

16. What value(s) of *v* satisfy $6v + 4 = 9v - 8$?

This becomes $12 = 3v$, giving $4 = v$.

17. Simplify by combining like terms: $7u - 3 - 6u^2 + 1 - 3u + 3 - 2u^2 + 3$

$7u - 3 - 6u^2 + 1 - 3u + 3 - 2u^2 + 3 = -8\mathrm{u}^2 + 4u + 4$

18. Olaf & Sven see one another when they are 280 meters apart and immediately start running towards one another. If Sven runs at 4 meters per second and Olaf runs at 3 meters per second, how many seconds will it take them to reach each other?

They approach one another at a combined speed of 7 mps, so it will take them $\frac{280}{7} = 40$ to meet.

19. In which quadrant does the point $(-7, -9)$ lie?

This point is in the lower left quadrant, which is the third.

20. What is the equation, in slope-intercept form $(y = mx + b)$, of the line through the points $(-3, 2)$ and $(-5, 6)$?

The slope is $m = \frac{2-6}{-3-(-5)} = \frac{-4}{2} = -2$ so the equation of the line is $y = -2x + b$. Substituting the first point gives $2 = -2(-3) + b$, then $2 = 6 + b$, and finally $b = -4$, for an answer of $y = -2x - 4$.

21. What is the slope of the line perpendicular to the line $4x + 2y = 8$?

The slope of this line is $m = -\frac{A}{B} = -\frac{4}{2} = -2$, so the slope of the perpendicular line will be $-\frac{1}{-2} = \frac{1}{2}$.

22. What is the distance between the points $(-3, 5)$ and $(3, -5)$?

The Distance Formula gives
$\sqrt{(-3-3)^2 + (5-(-5))^2} = \sqrt{6^2 + 10^2} = 2\sqrt{3^2 + 5^2} = 2\sqrt{9 + 25} = 2\sqrt{34}$.

23. The point $(-1, 1)$ is reflected across the line $x = -7$ to point K. What are the coordinates, in the form (x, y), of point K?

$(-1,1)$ is $-1 - (-7) = -1 + 7 = 6$ units to the right of $x = -7$, so the new point will be 6 units to the left of $x = -7$ at $(-13,1)$.

24. What is the shortest distance from the point $(0, 1)$ to the line $9x - y = -6$?

The line can be written as $9x - y + 6 = 0$, so that the distance of the point from the line will be $\frac{|9\cdot 0-1+6|}{\sqrt{9^2+1^2}} = \frac{|0-1+6|}{\sqrt{81+1}} = \frac{5}{\sqrt{82}} = \frac{5\sqrt{82}}{82}$.

25. What is the equation of the axis of symmetry of the parabola $x = 8y^2 - 7y - 5$?

This parabola is sideways! Thus, its axis of symmetry will be $y = -\frac{b}{2a} = -\frac{-7}{2\cdot 8} = \frac{7}{16}$.

26. Katie glues a rectangular picture measuring 9 cm by 26 cm to a rectangular piece of paper so that there is 8 cm of paper showing on each side of the picture. What is the total area, in square centimeters, of the paper that is showing around the edges of the picture?

The perimeter of the picture is $9 + 26 + 9 + 26 = 35 + 35 = 70$, and there is a rectangle extending 8 cm from each side, so the combined areas of these rectangles will be $8 \cdot 70 = 560$. In addition there are four 8 cm squares in the corners, with a combined area of $4 \cdot 8^2 = 16^2 = 256$, giving an answer of $560 + 256 = 816$.

27. If you can buy S gallons of syrup for D dollars, how many cents would it cost to buy four gallons of syrup?

The cost in dollars for one gallon of syrup is $\frac{D}{S}$. Thus, four gallons would cost $\frac{4D}{S}$ dollars. In cents, that would be $\frac{400D}{S}$.

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28. The average age of Nadia, Ori, and James is 8. What will their average age be in four years?

It doesn't matter how old they are right now, in four years they will each be four years older, so their average age will also go up by four, for an answer of $8 + 4 = 12$.

29. I have twelve coins in my pocket worth a total of 52 cents. If the only possible coins are nickels, dimes, and pennies, and there is at least one of each of those, how many dimes are there?

There must be 2, 7, or 12 pennies. Obviously 12 pennies doesn't work. Seven pennies leaves five coins for a total of 45 cents, which would be four dimes and a nickel for an answer of 4. Two pennies would leave 10 coins for a total of 50 cents, which would be 10 nickels and no dimes, which isn't allowed, so the answer is 4.

30. What value(s) of $\boldsymbol{h}$ satisfy $\frac{h+7}{h+1} = \frac{h-3}{h-1}$?

Cross-multiplying gives $h^2 + 6h - 7 = h^2 - 2h - 3$, which becomes $8h = 4$, giving $h = \frac{1}{2}$.

31. Simplify by expanding and combining like terms: $(9j - 8)^2$

FOIL gives $81j^2 - 72j - 72j + 64 = 81j^2 - 144j + 64$.

32. If $k(m) = 6m + 3m^2$, evaluate $k(-3)$.

$k(-3) = 6(-3) + 3(-3)^2 = -18 + 3 \cdot 9 = -18 + 27 = 9$

33. If $n \blacksquare p = np - \frac{n}{p}$, evaluate $8 \blacksquare 4$.

$8 \blacksquare 4 = 8 \cdot 4 - \frac{8}{4} = 32 - 2 = 30$

34. A right triangle has legs measuring 7 m and 3 m. What is the length, in meters, of its hypotenuse?

The Pythagorean Theorem gives $\sqrt{7^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58}$.

35. What is the smallest possible perimeter, in meters, of an isosceles triangle with sides measuring 6 m and 17 m?

You can't have a 6-6-17 triangle, because $6 + 6 = 12 < 17$, so it must be a 6-17-17 triangle with a perimeter of $6 + 17 + 17 = 40$.

36. What is the area, in square meters, of an equilateral triangle with sides measuring 6 m?

You can break it into 30-60-90 triangles, or know the formula $A = \frac{s^2\sqrt{3}}{4} = \frac{6^2\sqrt{3}}{4} = 3^2\sqrt{3} = 9\sqrt{3}$.

37. A right triangle with an angle measuring $45°$ has a hypotenuse measuring 6 m. What is the length, in meters, of a leg of the right triangle?

This is a 45-45-90 triangle, whose hypotenuse is $\sqrt{2}$ times each of its legs, for an answer of $\frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$.

38. What is the name of the point where the three medians of a triangle meet?

This is the "centroid" or "center of mass".

39. What is the name for a polygon with eight sides?

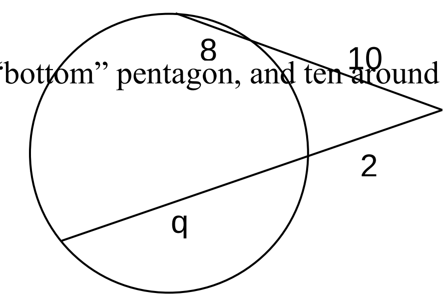
You should have memorized "octagon".

40. Two similar rhombi have perimeters of 60 m and 40 m. If the smaller rhombus has an area of 30 m^2, what is the area, in square meters, of the larger rhombus?

The areas will be in the ratio of $\left(\frac{60}{40}\right)^2 = \left(\frac{6}{4}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$, for an answer of $\frac{9}{4} \cdot 30 = \frac{9}{2} \cdot 15 = \frac{135}{2}$.

41. How many vertices does a regular dodecahedron have?

There are five around the "top" pentagon, five around the "bottom" pentagon, and ten around the "middle", for a total of $5 + 5 + 10 = 20$.



42. The figure to the right shows two secant line segments intersecting a circle, with all segment lengths given in meters. What is the value of q?

The formula is $10(10 + 8) = 2(2 + q)$, which becomes $180 = 2(2 + q)$, then $90 = 2 + q$, and finally $q = 88$.

43. What is the name for the line segment from a vertex of a triangle perpendicular to the line containing the opposite side of the triangle? Note: sometimes this line segment lies outside the triangle.

This is the "altitude".

44. A right triangle has legs measuring 3 m and 4 m. What is the length, in meters, of the altitude to its hypotenuse?

We know the hypotenuse is 5, and considering the area, it could be based on $3 \cdot 4$ or $a \cdot 5$, so we can write $5a = 12$, so $a = \frac{12}{5}$.

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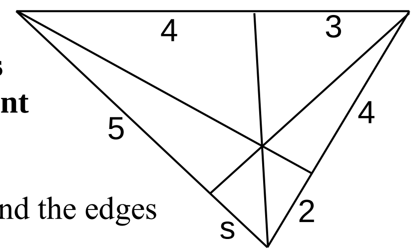
45. A triangle has two sides measuring 9 m and 16 m. What is the largest integer that could be the length in meters of the third side of the triangle?

The two sides could add together to make a length of 25, but in that case the third side would lie on the same line as the other two sides, so the longest the third side can be is 24.

46. What is the perimeter, in meters, of a square that is inscribed in a circle with a circumference of 7π?

If the circumference is 7π, the diameter is 7, which is the diagonal of the square. Each side of the square is thus $\frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$, so its perimeter will be $14\sqrt{2}$.



47. The figure to the right shows a triangle with three cevians drawn through an interior point, and all perimeter segment lengths are given in meters. What is the value of s?

Ceva's Theorem says that the product of the three ratios around the edges will be $1 = \frac{2}{4} \cdot \frac{3}{4} \cdot \frac{5}{s}$, which gives $s = \frac{2 \cdot 3 \cdot 5}{4 \cdot 4} = \frac{30}{16} = \frac{15}{8}$.

48. What is the measure, in degrees, of an interior angle of a regular pentagon?

The sum of the interior angles of a pentagon is $(5-2) \cdot 180 = 3 \cdot 180 = 540$, so each angle will be $\frac{540}{5} = 108$.

49. What is the volume, in cubic meters, of a right circular cone with a height of 3 m and a base radius of 7 m?

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 7^2 \cdot 3 = 49\pi$

50. A goat is tethered to an exterior corner of a rectangular shed surrounded by blackberries. The shed measures 2 m by 9 m, and the goat's chain is 6 m long. What is the area, in square meters, of the area the goat can clear of blackberries?

The goat can graze $\frac{3}{4}$ of a circle of radius 6, and additionally can wrap around one corner to do $\frac{1}{4}$ of a circle of radius 4, for an answer of $\frac{3}{4} \cdot 6^2\pi + \frac{1}{4} \cdot 4^2\pi = 3 \cdot 3^2\pi + 2^2\pi = 31\pi$.

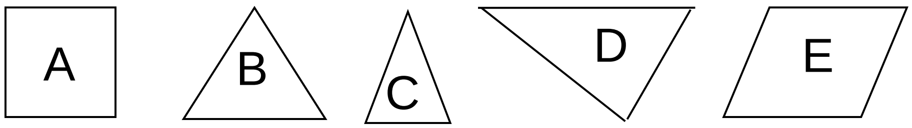
51. Two concentric circles are drawn, and a chord of the larger circle is drawn that happens to be tangent to the smaller circle. If the chord measures 8 m, what is the area, in square meters, of the annular region between the two circles?

Drawing the figure, you'll realize that half the chord is a leg of a right triangle with hypotenuse R and other leg r, so it satisfies $R^2 = r^2 + 4^2 = r^2 + 16$. The answer we're looking for is $\pi(R^2 - r^2)$, which is simply 16π.

52. How many diagonals can be drawn in a convex 15-gon?

There are 15 places a diagonal could start, and $15 - 3 = 12$ places that diagonal could end, for a potential answer of $15 \cdot 12 = 180$. However, we must realize that we have counted each diagonal from both ends, so we must divide our answer by 2 to get 90.

53. List the letters of the shapes below that can tessellate (completely cover, without overlapping) a plane by themselves.



D and E can both tessellate a plane, so A, B, and C obviously will as special cases of D and E, making the answer ABCDE.

54. What is the measure, in degrees, of an angle complementary to one measuring $37°$?

Complementary angles add up to a right angle, so our answer is $90 - 37 = 53$.

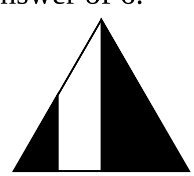
55. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 5:30 AM?

At 5:30, the minute hand will point at the 6 and the hour hand will be halfway between the 5 and the 6. The angle between them will thus be half the angle between 5 and 6, or $\frac{1}{2} \cdot \frac{1}{12} = \frac{1}{24}$ of the entire clock face, for an answer of $\frac{360}{24} = \frac{180}{12} = \frac{90}{6} = 15$.

56. A cube of white plastic measures 9 cm on all edges and is painted blue on all faces. When this cube is cut into smaller cubes measuring 3 cm on all edges, how many of those smaller cubes have exactly one blue face?

The cube is cut into $\frac{9}{3} = 3$ slices in each of the three dimensions, for a total of $3^3 = 27$ small cubes. To get a cube with just one blue face, it must be in the interior of a face of the large cube. In this case, there is just one such cube on each of the six faces, for an answer of 6.

57. What fraction of the equilateral triangle to the right is shaded? The unshaded region is bordered by two line segments perpendicular to the base, one of which is twice as long as the other.



Half the triangle is shaded, as is a quarter of the remaining half, which is $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$.

58. How many real roots does $8n^2 + 9n + 1 = 0$ have?

The discriminant is $b^2 - 4ac = 9^2 - 4 \cdot 8 \cdot 1 = 81 - 32 > 0$, so there will be two roots.

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59. What are the coordinates, in the form (x, y), of the center of the circle with equation $x^2 + y^2 - 2x + 3y = 21$?

Completing the square gives $(x-1)^2 + \left(y + \frac{3}{2}\right)^2 = \cdots$, for an answer of $\left(1, -\frac{3}{2}\right)$.

60. What is the area of the ellipse with equation $\frac{(x-2)^2}{15} + \frac{(y+5)^2}{9} = 1$?

The "width radius" of this ellipse will be $\sqrt{15}$, and it's "height radius will be $\sqrt{9} = 3$, for an answer of $A = \pi ab = \pi \cdot \sqrt{15} \cdot 3 = 3\pi\sqrt{15}$.

61. Evaluate: $\log_2 32$

Because $2^5 = 32$, $\log_2 32 = 5$.

62. What value(s) of t satisfy $3^{2t} - 18 = 7 \cdot 3^t$?

This is a hidden quadratic equation, which we can write as $3^{2t} - 7 \cdot 3^t - 18 = 0$ and factor to $(3^t - ?)(3^t + ?) = 0$, which we can determine to be $(3^t - 9)(3^t + 2) = 0$. The first factor will be zero if $t = 2$, and the second factor will never be zero, so our answer is just 2.

63. If \$1000 is invested at 10% interest compounded annually, what will its value be after two years? Express your answer in dollars rounded to the nearest hundredth (cent).

After the first year we'll earn \$100, giving us a total of \$1100. After the second year we'll earn \$110, for an answer of \$1210.

64. What is the sum of the roots of $7u^3 + 6u^2 + 8u - 5 = 0$?

The sum of the roots of a polynomial is always $-\frac{b}{a} = -\frac{6}{7}$.

65. What is the sum of the prime numbers between 45 and 55?

47 is prime, 49 is not, 51 is not $(3 \cdot 17)$, and 53 is, for an answer of $47 + 53 = 100$.

66. Express the base 10 number 699_{10} as a base 7 number.

In base seven, the digits from right to left represent $7^0 = 1$, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, etc. 699 contains two 343s, leaving $699 - 686 = 13$, so there are no 49s, one 7, and six 1s, for an answer of 2016.

67. Express the base three numeral 120210_3 as a base nine numeral.

Every two digits in base three represents a digit in base nine. $10_3 = 3_9$, $02_3 = 2_9$, $12_3 = 5_9$, for an answer of 523_9.

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68. What is the smallest value of $c > 100$ satisfying $c \equiv 3 \ mod\ 30$?

The numbers congruent to $3\ mod\ 30$ are 3, 33, 63, 93, 123, 153, etc., making 123 the answer.

69. How many positive integers are factors of 40?

$40 = 2^3 \cdot 5^1$, so it has $4 \cdot 2 = 8$ factors.

70. What is the sum of the positive integer factors of 48?

$48 = 2^4 \cdot 3^1$, so the sum of its factors will be $(1 + 2 + 4 + 8 + 16)(1 + 3) = 31 \cdot 4 = 124$.

71. An 8-by-12 array of unit squares is drawn, as is a line from the upper left corner to the lower right corner of the array. How many of the unit squares does the line pass through?

Because 8 and 12 have a greatest common factor of 4, the diagonal line will pass through four regions that each measure 2-by-3, transitioning from one to the next at their corners. Within each of these regions, the line will pass through four squares, so the total number of squares will be $4 \cdot 4 = 16$.

72. What is the 24th term of an arithmetic sequence with first term 5 and common difference 6?

The 24th term will be 23 differences from the 1st term, so will be $5{+}23 \cdot 6 = 5 + 138 = 143$.

73. What is the third term of a harmonic sequence with first term 6 and second term 4?

A harmonic sequence has elements that are the reciprocals of an arithmetic sequence. The reciprocals of these terms are $\frac{1}{6}$ and $\frac{1}{4}$. Their difference is $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$, so the next reciprocal in arithmetic progression will be $\frac{1}{4} + \frac{1}{12} = \frac{1}{3}$, for an answer of 3.

74. What is the missing term of the sequence 3, 2, 6, 10, __, 18, 24, 26, 48, 34, …?

Confusing sequences like this are often two or more interspersed sequences. In this case we have the arithmetic sequence 2, 10, 18, 26, 34 interspersed with the geometric sequence 3, 6, 12, 24, 48, making the answer 12.

75. Evaluate: $\sum_{d=1}^{8}\left(\frac{1}{d} - \frac{1}{d+1}\right)$

This is $\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{6} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{9}\right)$, which has a lot of cancellation, and becomes $\frac{1}{1} - \frac{1}{9} = \frac{8}{9}$.

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76. What is the sum of the six smallest positive even numbers?

This should be twice as large as the sum of the six smallest whole numbers. For that problem, consider the "outer pairs" $1+6=7$, $2+5=7$, and $3+4=7$. We end up with $\frac{6}{2}=3$ pairs that each sum to 7, for a total of $3\cdot 7=21$. Our answer is thus $2\cdot 21=42$.

77. What is the sum of the six smallest perfect squares?

The formula for this is $\frac{n(n+1)(2n+1)}{6}=\frac{6\cdot 7\cdot 13}{6}=7\cdot 13=91$.

78. What is the sum of the eight smallest positive perfect cubes?

The formula for this is $\left(\frac{n(n+1)}{2}\right)^2=\left(\frac{8\cdot 9}{2}\right)^2=(4\cdot 9)^2=36^2=1296$.

79. When two marbles are drawn from a bag containing 8 red and 4 blue marbles, what is the probability that they are different colors?

There are $8c1=8$ ways to get a red and $4c1=4$ ways to get a blue, and $12c2=\frac{12!}{10!2!}=\frac{12\cdot 11}{2}=6\cdot 11=66$ ways to draw two marbles in general, for a probability of $\frac{8\cdot 4}{66}=\frac{4\cdot 4}{33}=\frac{16}{33}$.

80. When a single card is drawn from a standard 52-card deck, what is the probability that it is either a heart or a face card (a King, Queen, or Jack)?

There are 13 hearts and $3\cdot 3=9$ other face cards, for a total of $13+9=22$ cards we could draw and a probability of $\frac{22}{52}=\frac{11}{26}$.

81. A trusted friend flips four coins behind a screen and tells you that there are at least two heads showing. What is the probability there are at least three heads showing?

There are $4c0=1$ way to get no heads, $4c1=4$ ways to get one head, $4c2=6$ ways to get two heads, $4c3=4$ ways to get three heads, and $4c4=1$ way to get four heads. As it turns out, we did not get no heads or one head, so there are only $6+4+1=11$ ways things could have happened. Of those, $4+1=5$ ways are what we're looking for, so the answer is $\frac{5}{11}$.

82. When five people run a race, in how many ways can a first-place trophy and a second-place medal be awarded?

There are five people who could get the trophy, after which there are only four people who could get the medal, for an answer of $5\cdot 4=20$.

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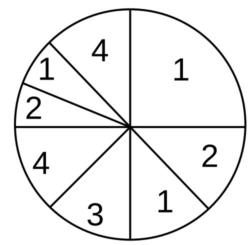
83. In how many ways can the letters of "PAPAYA" be arranged?

There are 6 letters, so 6! is sort of the answer. There are three A's, however, so they can be arranged in 3! indistinguishable ways, and there are two P's, which can be arranged in 2! indistinguishable ways, for an answer of $\frac{6!}{3!2!} = \frac{6\cdot5\cdot4}{2} = 3 \cdot 5 \cdot 4 = 60$.

84. In the game of Sparc, a player pays \$5 to roll a single die, and is paid $(n-2)^2$ dollars if the die showed n on top. What is the expected value, rounded to the nearest whole number of cents, of the person's profit when they play this game once?

You'll be payed 1, 0, 1, 4, 9, or 16 dollars, each with a probability of $\frac{1}{6}$, so the expected payment is $\frac{1+1+4+9+16}{6} = \frac{31}{6}$, and the expected profit is $\frac{31}{6} - 5 = \frac{1}{6}$ dollars, which is $16.\overline{6}$ cents, for an answer of 17.

85. When playing her favorite board game, what is the probability that Katie gets a 4 when she spins the spinner shown to the right? Assume that central angles are always produced by dividing by two.



Each 4 has a $\frac{1}{8}$ probability, making the total probability $\frac{1}{4}$.

86. James wants to wear three matching earrings today, but needs to select them in the dark while his wife is sleeping. If he knows that his earring bowl contains five diamond studs, eleven ruby studs, and nine emerald studs, what is the minimum number of earrings he can take and be certain that he will have three matching earrings?

The worst case is that James gets 2 of each type, for a total of 6. At this point, no matter what he selects he gets a set of three and is happy, so the answer is 7.

87. What is the median of the data set $\{48, 83, -79, 96, 818, 382, 30\}$?

The median is the central element when the elements are arranged in numerical order, which you can find by simply eliminating pairs of high and low elements. The set can be rearranged $\{-79, 30, 48, 83, 96, 382, 818\}$, after which we can eliminate -79 & 818, 30 & 382, and 48 & 96 to leave 83.

88. What is the mode of the data set {91, 69, 97, 32, -25, 36, -57, 97}?

The mode is the most common element, so 97 (there are two of them).

89. What is the range of the data set {-91, -54, -18, 72, -74, 56, 425, -48, -6}?

The range is the largest element minus the smallest element, so
$425 - (-91) = 425 + 91 = 516$.

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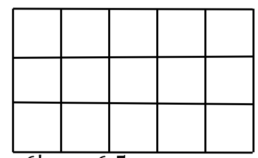
90. Evaluate: $< \mathbf{1,4,5} > \cdot < \mathbf{9,5,7} >$

The dot product of these vectors is
$< 1,4,5 > \cdot < 9,5,7 > = 1 \cdot 9 + 4 \cdot 5 + 5 \cdot 7 = 9 + 20 + 35 = 64$

91. How many subsets of {2, 3, 5, 8, 13, 21} contain exactly two odd numbers?

There are four odd numbers, and we want two of them; there are $4c2 = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2} = 2 \cdot 3 = 6$ ways to choose them. We don't care about even numbers; we can take them or leave them. Thus, we have two choices about what to do with the 2 and two choices about what to do with the 8, for an answer of $6 \cdot 2 \cdot 2 = 24$.



92. In the array of unit squares to the right, how many rectangles of any size or shape can be drawn along the gridlines?

Any two vertical lines can be the sides of a rectangle, and there are $6c2 = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2} = 3 \cdot 5 = 15$ ways to choose them. The top & bottom can be chosen in $4c2 = 6$ ways, for an answer of $15 \cdot 6 = 90$.

93. What is the area, in square meters, of a triangle with sides measuring 5 m, 7 m, and 8 m?

Heron's Formula gives $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10 \cdot 5 \cdot 3 \cdot 2} = 10\sqrt{3}$.

94. What is the period, in radians, of the function $\boldsymbol{k(m) = 2\sin(3m) + 4\cos(5m)}$?

This problem's not as interesting as we'd originally intended, but maybe that makes it a trick problem? The period of the sine is $\frac{2\pi}{3}$, and the period of the cosine is $\frac{2\pi}{5}$, for a combined period of 2π.

95. Express $\boldsymbol{4e^{-\frac{i\pi}{3}}}$ in standard $\boldsymbol{(a+bi)}$ form.

$e^{ix} = \cos x + i \sin x$, so $4e^{-\frac{i\pi}{3}} = 4\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{pi}{3}\right)\right) = 4\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = 2 - 2i\sqrt{3}$.

96. What is the most specific name for the shape of the locus of points satisfying $\boldsymbol{x = \sin t}$ and $\boldsymbol{y = 2\cos t}$?

If the coefficients of these parametric equations were the same, the answer would be "circle", because they satisfied $x^2 + y^2 = 1$, but with the different coefficients the answer is "ellipse", because it's "taller" than that circle.

97. If $\boldsymbol{n(p) = 3p^4}$, evaluate $\boldsymbol{\frac{dn}{dp}}$ when $\boldsymbol{p = 2}$.

The power rule gives $n'(p) = 12p^3$, so $n'^{(2)} = 12 \cdot 2^3 = 12 \cdot 8 = 96$.

98. If $q(r) = (5r - 4)(3 + 2r)$, evaluate $\frac{dq}{dr}$ when $r = -1$.

For simple cases like this, you could FOIL and use the power rule, but the product rule will definitely be faster for complex cases. $q'(r) = (5r - 4) \cdot 2 + (3 + 2\text{r}) \cdot 5 = 10\text{r} - 8 + 15 + 10\text{r} = 20\text{r} + 7$. $q'(-1)$ will thus be $20(-1) + 7 = -13$.

99. Evaluate $\lim_{s\to 3} \frac{s^2-9}{3s-9}$

Plugging in gives $\frac{0}{0}$, so we'll need to use some kind of trick. Once you know L'Hopital's Rule, it's almost always the easiest way to go, but the way you're "supposed" to do this one before then is factoring. $\lim_{s\to 3} \frac{s^2-9}{3s-9} = \lim_{s\to 3} \frac{(\text{s}+3)(\text{s}-3)}{3(s-3)} = \lim_{s\to 3} \frac{\text{s}+3}{3} = \frac{3+3}{3} = \frac{6}{3} = 2$.

100. Evaluate: $\int_1^2 e^{3w} dw$

The integral of e^x is e^x, but the chain rule in our case makes the integral $\frac{1}{3} e^{3w} \Big|_1^2 = \frac{1}{3} e^6 - \frac{1}{3} e^3$.