2016 Fall Startup Event
Thursday, September 29th, 2016

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate: $5608 - 2650$
2. What is the remainder when 4725 is divided by 6?
3. Evaluate: $-7 - 2 - (-5) - 5(-8)$
4. Round 4680.12568 to the nearest thousand.
5. Evaluate: 2^8
6. In how many ways can three objects be chosen from a group of nine objects?
7. Evaluate: $1 + 6 \times 3^2 - 5 + 2(7 + 3 - 8)^5$
8. Express in simplest radical form: $\sqrt[3]{192}$
9. What is the sum of the number of vertices on a nonagon, the number of edges on a tetrahedron, and the number of hours in a week?
10. Express $.\overline{39}$ as a fraction.
11. What value(s) of z satisfy $9z - 35 = 73$?
12. Simplify by combining like terms: $5y^2 - 6y + y^2 + 6 - 9y - 3y^2 + 5$
13. If three horses can eat four bales of hay in five days, how many bales of hay could one horse eat in thirty days?
14. How many **minutes** would it take me to bike forty miles at a speed of 15 miles per hour?
15. Two runners start at the same time from the same position on a quarter-mile track, running the same direction. If their speeds are 7 and 9 miles per hour, how many minutes will it take the faster runner to first pass the other runner?
16. If two numbers sum to 56 and differ by 38, what is the smaller of the two numbers?
17. What is the equation, in slope-intercept ($y = mx + b$) form, of the line through the points $(9, -3)$ and $(1,13)$?
18. What are the coordinates, in the form (x, y), of the x-intercept of the line $4x - 6y = 84$?

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19. What is the distance between the points $(71,35)$ and $(95,67)$?

20. What is the midpoint of the line segment connecting the points $(62,45)$ and $(356,47)$?

21. What is the shortest distance from the point $(7,-3)$ to the line $x+y=1$?

22. What are the coordinates, in the form (x,y), of the vertex of the parabola with equation $y=2x^2+8x-3$?

23. When the digits of a positive two-digit number are reversed, the new positive two-digit number is 27 less than the original number. What is the smallest possible value of the original number?

24. An 8-inch by 10-inch rectangular picture is glued to a larger rectangular piece of paper which extends three inches on all sides of the picture. What is the area, in square inches, of the paper that is showing?

25. If three Vectors are equivalent to five Ukuleles, how many Ukuleles would be equivalent to 120 Vectors?

26. Eight coins are worth a total of 38 cents. If there are only pennies, nickels, and dimes, how many nickels are there?

27. What is the solution, in the form (r,s,t), of the system of equations $r+s=3$, $s+t=5$, and $r+t=6$?

28. What value(s) of q satisfy $\frac{q-1}{q+1}=\frac{q-2}{q+3}$?

29. If $p(n)=3n-7$, evaluate $p(9)$.

30. What is the length, in meters, of the hypotenuse of a right triangle with legs measuring 4 m and 6 m?

31. One angle of an isosceles triangle measures 98°. What is the measure, in degrees, of another angle in the triangle?

32. What is the most specific name that can be applied to every triangle without any congruent angles?

33. What is the most specific name that can be applied to every quadrilateral with exactly two congruent sides?

34. What is the perimeter, in meters, of a heptagon with sides measuring 9 m?

35. What is the area, in square meters, of a circle circumscribed about a square with a perimeter of 24 m?

36. What is the sum of the angles in a pentagon?

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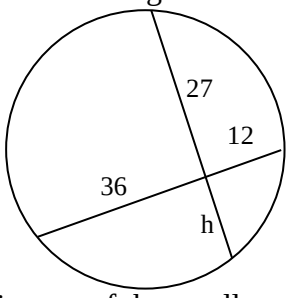
37. Two similar pentagons have perimeters of 12 m and 16 m. If the larger pentagon has an area of 32 m^2, what is the area, in square meters, of the smaller pentagon?

38. What is the volume, in cubic meters, of a cone with a base radius of 5 m and a height of 6 m?

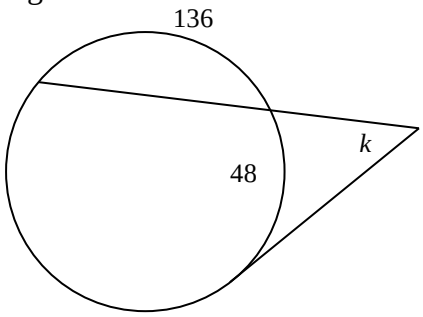
39. How many vertices are there on a dodecahedron?

40. The diagram to the right shows a circle with two intersecting chords, with segment lengths given in meters. What is the value of *h*?



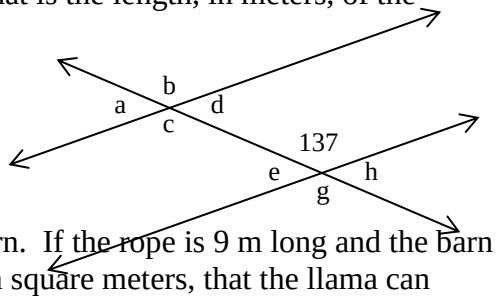
41. A triangle has sides measuring 6 m, 8 m, and 12 m. When the angle bisector of the smallest angle is drawn, what is the length, in meters, of the smaller segment into which it divides the opposite side?

42. The diagram to the right shows a circle with intersecting secant and tangent line segments, with angles and arcs measured in degrees. What is the value of *k*?



43. A right triangle has legs measuring 8 m and 10 m. What is the length, in meters, of the altitude to the hypotenuse?

44. The diagram to the right shows two parallel lines intersected by a third line, with angles measured in degrees. What is the value of $a + d + h$?



45. A llama is tethered to an outside corner of a square barn. If the rope is 9 m long and the barn has sides measuring 5 m each, what is the total area, in square meters, that the llama can graze?

46. Two circles with radii of 10 m have their centers 29 m apart. What is the length of a line segment between the two circles that is tangent to each circle and passes between them?

47. How many diagonals can be drawn in a convex 14-gon?

48. What is the largest number of regions into which a plane can be divided by two lines and a triangle?

49. What is the measure, in degrees, of an angle which is complementary to a 16° angle?

50. A regular polygon has vertices labeled in clockwise order from A to N. When a line is drawn through vertex H and the center of the polygon, what other vertex does it pass through?

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51. A cube of white plastic is painted blue on all of its faces, then cut into 125 congruent cubes. How many of the smaller cubes have paint on exactly two faces?

52. Evaluate: $i(2-3i)+4i^5$

53. Given a point and a line, consider the set of all points that are twice as far from the point as they are from the line. What is the name of the conic section that these points are a part of?

54. What is the area inside the locus of points satisfying $\frac{(x-1)^2}{2^2}+\frac{(y-3)^2}{4^2}=1$?

55. Evaluate: $\log_4 4096$

56. If $b(c)=c^3-9$, evaluate $b^{-1}(334)$.

57. If $d(f)=9-\sqrt{87-6f}$ has a domain and range which are subsets of the real numbers, express the domain in interval notation.

58. If g is directly proportional to h and $g=24$ when $h=36$, what is g when $h=24$?

59. What is the sum of the roots of $3j^3-2j^2+j-6=0$?

60. What is the coefficient of the k^3 term when $(2k-3)^5$ is expanded and like terms are combined?

61. Evaluate: $\left(\frac{25}{9}\right)^{\frac{3}{2}}$

62. Express the base ten numeral 123_{10} as a base three numeral.

63. What is the prime factorization, in exponential form, of 594?

64. How many positive integers are factors of 603?

65. What is the sum of the positive integer factors of 360?

66. What is the least common multiple of 45 and 80?

67. How many positive four-digit integers are composed of four different digits?

68. What is the sum of the numbers that are multiples of five in the list 82, 1465, 63, 75, 780?

69. What is the units digit of 987^{654}?

70. What is the seventh term of a geometric sequence with first term 6 and common ratio 2?

71. What is the missing term of the sequence 2, 4, 6, 28, 18, 52, 54, __, 162, 100?

72. What is the 37th term of an arithmetic sequence with first term 47 and common difference 17?

73. What is the sum of the 24 smallest positive integers?

74. What is the sum of the 13 smallest positive odd integers?

75. What is the sum of the 15 smallest positive perfect squares?

76. What is the sum of the 8 smallest positive perfect cubes?

77. When four fair coins are flipped, what is the probability that exactly two of them show tails?

78. A trusted friend rolls two standard six-sided dice behind a screen and declares that she did not roll doubles. What is the probability she rolled a sum of 7?

79. The probability that it rains tomorrow is $\frac{1}{4}$, and the probability that I read a book tomorrow is $\frac{2}{3}$. If these events are independent, what is the probability that it rains but I do not read a book?

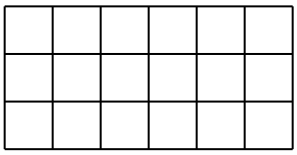
80. In how many ways can the letters in the word "HORROR" be arranged?

81. In how many ways can seven people sit relative to one another at a round table?

82. At my family reunion, 37 people liked hot dogs and 46 people liked hamburgers. If 13 people liked both and 31 liked neither, how many people were at the family reunion?

83. I pick two random real numbers from 1 to 5 and plot a point on the Cartesian plane using the first number as the x-coordinate and the second as the y-coordinate. What is the probability that this point satisfies $y > 2x$?

84. In the grid of unit squares to the right, how many paths of length 9 are there from the top left corner to the bottom right corner?



85. In how many ways can 8 identical candies be distributed among four children if fairness is not considered?

86. What is the range of the data set {4, 71, 58, 60, 5, 13, 61, 35, 70}?

87. Set C is {70, 5, 93, 57, 3} and Set D is {57, 90, 71, 3, 683}. Write the set $C \cap D$.

88. Set E is {5, 6, 7, 93} and Set F is {93, 5, 7}. How many subsets of E are supersets of F?

89. If $G = \begin{bmatrix} 9 & 4 \\ 4 & 2 \end{bmatrix}$, what is G^{-1}?

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90. In the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$, what is the cofactor of the 6?

91. In the cross-number puzzle to the right, each of U, V, W, and X is a positive one-digit integer collectively satisfying four equations: two horizontal, two vertical. What is the product of U, V, W, and X?

U	x	V	=	8
-		+		
W	+	X	=	8
=		=		
3		9		

92. Evaluate: $\frac{1}{4} \cdot \frac{2}{5} \cdot \frac{3}{6} \cdots \frac{10}{13} \cdot \frac{11}{14} \cdot \frac{12}{15}$

93. In a right triangle with legs measuring 3 m and 5 m, what is the tangent of the smallest angle?

94. Express $216°$ in radians.

95. What is the period of $y = 7\cos(6x - 5) + 4$?

96. Evaluate the function: $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

97. If $m(t) = 3t^4$, evaluate $m'(-2)$.

98. Evaluate: $\int_1^2 (n(n^2 - 3)^4) dn$

99. If the velocity of a particle on a line is $v(t) = 3e^{2t} + 1$, what is the acceleration of the particle when $t = -1$?

100. The velocity of a particle moving along a line is graphed to the right on a unit grid. At what value(s) of *t* does the particle change direction?