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1. Evaluate: $\mathbf{5608 - 2650}$

The standard algorithm gives 2958.

2. What is the remainder when 4725 is divided by 6?

4725 is odd (ends in an odd number) and a multiple of 3 (digits sum to 18, a multiple of 3), so it must leave a remainder of 3 when divided by 6.

3. Evaluate: $\mathbf{-7 - 2 - (-5) - 5(-8)}$

$-7 - 2 - (-5) - 5(-8) = -7 - 2 + 5 + 40 = -9 + 45 = 36$

4. Round 4680.12568 to the nearest thousand.

4,680 rounds up to 5,000.

5. Evaluate: $\mathbf{2^8}$

$2^8 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 4 \cdot 4 \cdot 4 \cdot 4 = 16 \cdot 16 = 256$

6. In how many ways can three objects be chosen from a group of nine objects?

$9c3 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = 3 \cdot 4 \cdot 7 = 84$

7. Evaluate: $\mathbf{1 + 6 \times 3^2 - 5 + 2(7 + 3 - 8)^5}$

$$\begin{aligned} 1 + 6 \times 3^2 - 5 + 2(7 + 3 - 8)^5 &= 1 + 6 \times 3^2 - 5 + 2 \cdot 2^5 = 1 + 6 \times 9 - 5 + 2 \cdot 32 \\ &= 1 + 54 - 5 + 64 = 114 \end{aligned}$$

8. Express in simplest radical form: $\mathbf{\sqrt[3]{192}}$

$\sqrt[3]{192} = \sqrt[3]{8 \cdot 24} = 2\sqrt[3]{24} = 2\sqrt[3]{8 \cdot 3} = 2 \cdot 2\sqrt[3]{3} = 4\sqrt[3]{3}$

9. What is the sum of the number of vertices on a nonagon, the number of edges on a tetrahedron, and the number of hours in a week?

A nonagon has nine vertices, a tetrahedron has four *faces* but six edges, and there are $7 \cdot 24 = 168$ hours in a week, for an answer of $9 + 6 + 168 = 183$.

10. Express $\mathbf{.\overline{39}}$ as a fraction.

If we write $x = .\overline{39}$, we can also write $100x = 39.\overline{39}$. Subtracting these two gives $99x = 39$ and we've got rid of the pesky repeating decimal! Finally, we get $x = \frac{39}{99} = \frac{13}{33}$.

11. What value(s) of *z* satisfy $\mathbf{9z - 35 = 73}$?

Adding 35 to both sides gives $9z = 108$, and then dividing by 9 gives $z = 12$.

12. Simplify by combining like terms: $5y^2 - 6y + y^2 + 6 - 9y - 3y^2 + 5$

$5y^2 + y^2 - 3y^2 = 3y^2$, $-6y - 9y = -15y$, and $6 + 5 = 11$ gives an answer of $3y^2 - 15y + 11$.

13. If three horses can eat four bales of hay in five days, how many bales of hay could one horse eat in thirty days?

We've divided the number of horses by 3, so we need to divided the hay by 3. We've multiplied the days by 6, so we need to multiply the hay by 6. Together, this gives an answer of $\frac{4\cdot 6}{3} = 4 \cdot 2 = 8$.

14. How many minutes would it take me to bike forty miles at a speed of 15 miles per hour?

$d = rt$, or "dirt", is a way to remember that "distance equals rate times time", so time must be distance divided by rate, giving $\frac{40}{15} = \frac{8}{3}$ hours, which is $\frac{8}{3} \cdot 60 = 8 \cdot 20 = 160$.

15. Two runners start at the same time from the same position on a quarter-mile track, running the same direction. If their speeds are 7 and 9 miles per hour, how many minutes will it take the faster runner to first pass the other runner?

To pass the other runner, the faster runner must run one extra lap, which means they need to go $\frac{1}{4}$ mile farther than the slower runner. The faster runner goes $9 - 7 = 2$ miles per hour faster than the slower runner, so it will take them $\frac{\frac{1}{4}}{2} = \frac{1}{8}$ of an hour to pass the slower runner, which is $\frac{1}{8} \cdot 60 = \frac{30}{4} = \frac{15}{2}$ minutes.

16. If two numbers sum to 56 and differ by 38, what is the smaller of the two numbers?

The two numbers must average $\frac{56}{2} = 28$, and each must be $\frac{38}{2} = 19$ from that average, making the smaller number $28 - 19 = 9$.

17. What is the equation, in slope-intercept $(y = mx + b)$ form, of the line through the points $(9, -3)$ and $(1, 13)$?

The slope is the change-in-*y* over the change-in-*x*, $m = \frac{13-(-3)}{1-9} = \frac{16}{-8} = -2$. At this point we can write $y = -2x + b$, but we don't know the value of *b*. Substituting the first point gives $-3 = -2 \cdot 9 + b = -18 + b$, giving $b = -3 + 18 = 15$, for an answer of $y = -2x + 15$.

18. What are the coordinates, in the form (x, y), of the x-intercept of the line $4x - 6y = 84$?

The x-intercept is on the x-axis, which is where $y = 0$, so that $4x = 84$, which gives $x = 84 \div 4 = 21$ for an answer of $(21,0)$.

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19. What is the distance between the points $(71, 35)$ and $(95, 67)$?

The Distance Formula gives $\sqrt{(95-71)^2+(67-35)^2} = \sqrt{24^2+32^2} = 8\sqrt{3^2+4^2} = 8\sqrt{9+16} = 8\sqrt{25} = 8 \cdot 5 = 40$.

20. What is the midpoint of the line segment connecting the points $(62, 45)$ and $(356, 47)$?

The x-coordinate of the midpoint will be $\frac{356+62}{2} = \frac{418}{2} = 209$, and the y-coordinate will be $\frac{45+47}{2} = \frac{92}{2} = 46$, for an answer of $(209,46)$.

21. What is the shortest distance from the point $(7, -3)$ to the line $x + y = 1$?

It's fastest if you've memorized that you plug the point into the line (in $= 0$ form) and divide by $\sqrt{A^2+B^2}$, which gives $\frac{7+(-3)-1}{\sqrt{(1^2+1^2)}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$.

22. What are the coordinates, in the form (x, y), of the vertex of the parabola with equation $y = 2x^2 + 8x - 3$?

The axis of symmetry will be $x = -\frac{b}{2a} = -\frac{8}{2\cdot 2} = -\frac{8}{4} = -2$, so the y-coordinate of the vertex will be $y = 2(-2)^2 + 8(-2) - 3 = 8 - 16 - 3 = -11$, for an answer of $(-2, -11)$.

23. When the digits of a positive two-digit number are reversed, the new positive two-digit number is 27 less than the original number. What is the smallest possible value of the original number?

When a number such as 59 is reversed to become 95, the 5 tens become 5 ones, which is a loss of $5(10-1) = 5 \cdot 9 = 45$. Similarly, the 9 ones become 9 tens, which is a gain of $9(10-1) = 9 \cdot 9 = 81$, for a total gain of $81 - 45 = 9(9-5)$. In general, a reversed number will gain or lose nine times the difference of its digits. In this case, we want a loss of 27, so we need a difference of $27 \div 9 = 3$. 30 might be a good candidate, but when it's reversed it doesn't form a normal two-digit number, so 41 is the smallest such number. $41 - 14 = 27$.

24. An 8-inch by 10-inch rectangular picture is glued to a larger rectangular piece of paper which extends three inches on all sides of the picture. What is the area, in square inches, of the paper that is showing?

Each side has a 3-inch rectangle of paper beyond it, and there are four 3-inch squares in the corners. The area is thus $3(2 \cdot 8 + 2 \cdot 10) + 4 \cdot 3^2 = 3(16+20) + 4 \cdot 9 = 3 \cdot 36 + 36 = 4 \cdot 36 = 144$.

25. If three Vectors are equivalent to five Ukuleles, how many Ukuleles would be equivalent to 120 Vectors?

120 Vectors is $120 \div 3 = 40$ batches of 3 Vectors, so would be equivalent to 40 batches of 5 Ukuleles, for an answer of $40 \cdot 5 = 200$.

26. Eight coins are worth a total of 38 cents. If there are only pennies, nickels, and dimes, how many nickels are there?

At least three coins are pennies, leaving five coins worth 35 cents. There cannot be any more pennies, so the rest are two dimes and three nickels, for an answer of 3.

27. What is the solution, in the form (r, s, t), of the system of equations $r + s = 3$, $s + t = 5$, and $r + t = 6$?

Adding all three equations gives $2r + 2s + 2t = 3 + 5 + 6 = 14$, which is equivalent to $r + s + t = 7$. Subtracting each of the given equations from this gives $t = 7 - 3 = 4$, $r = 7 - 5 = 2$, and $s = 7 - 6 = 1$, for an answer of $(2,1,4)$.

28. What value(s) of q satisfy $\frac{q-1}{q+1} = \frac{q-2}{q+3}$?

Cross-multiplying gives $q^2 - q + 3q - 3 = q^2 - 2q + q - 2$, which becomes $2q - 3 = -q - 2$, then $3q = 1$, for an answer of $\frac{1}{3}$.

29. If $p(n) = 3n - 7$, evaluate $p(9)$.

$p(9) = 3 \cdot 9 - 7 = 27 - 7 = 20$

30. What is the length, in meters, of the hypotenuse of a right triangle with legs measuring 4 m and 6 m?

$\sqrt{4^2 + 6^2} = 2\sqrt{2^2 + 3^2} = 2\sqrt{4 + 9} = 2\sqrt{13}$

31. One angle of an isosceles triangle measures 98°. What is the measure, in degrees, of another angle in the triangle?

You cannot have a $98 - 98 - x$ triangle, so it must be a $98 - x - x$ triangle. To add up to 180, $2x = 180 - 98 = 82$, for an answer of 41°.

32. What is the most specific name that can be applied to every triangle without any congruent angles?

If the angles are different, the sides will be different, for an answer of “scalene”.

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33. What is the most specific name that can be applied to every quadrilateral with exactly two congruent sides?

Having two congruent sides is not enough to reliably make the quadrilateral interesting in any way, so "quadrilateral" is the most specific name you can use for all such shapes.

34. What is the perimeter, in meters, of a heptagon with sides measuring 9 m?

7 sides each measuring 9 gives a perimeter of $7 \cdot 9 = 63$.

35. What is the area, in square meters, of a circle circumscribed about a square with a perimeter of 24 m?

If the perimeter is 24, each side is $24 \div 4 = 6$. A diameter of the circle will be the diagonal of the square, which is $6\sqrt{2}$, so the circle's radius is $3\sqrt{2}$, for an area of $\pi \cdot \left(3\sqrt{2}\right)^2 = 9 \cdot 2 \cdot \pi = 18\pi$.

36. What is the sum of the angles in a pentagon?

Drawing diagonals from a single vertex divides a pentagon into three triangles, so the sum of the angles will be $3 \cdot 180 = 540°$.

37. Two similar pentagons have perimeters of 12 m and 16 m. If the larger pentagon has an area of 32 m^2, what is the area, in square meters, of the smaller pentagon?

The ratio between the dimensions of the two pentagons is $\frac{12}{16} = \frac{3}{4}$, so the ratio of the areas is $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$, making the smaller area $32 \cdot \frac{9}{16} = 2 \cdot 9 = 18$.

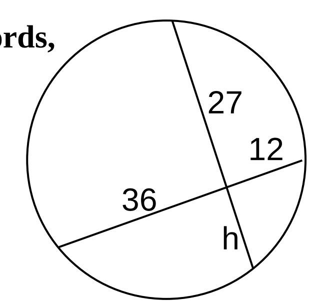
38. What is the volume, in cubic meters, of a cone with a base radius of 5 m and a height of 6 m?

$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 5^2 \cdot 6 = 2\pi \cdot 25 = 50\pi$

39. How many vertices are there on a dodecahedron?

A dodecahedron has twelve faces, each of which is a pentagon, so one might think there are $12 \cdot 5 = 60$ vertices. However, each vertex is part of three different pentagons, so the actual number of vertices is $60 \div 3 = 20$.

40. The diagram to the right shows a circle with two intersecting chords, with segment lengths given in meters. What is the value of *h*?

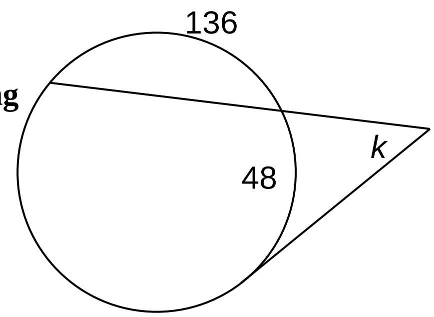


$27 \cdot h = 36 \cdot 12$, so $h = \frac{36 \cdot 12}{27} = \frac{4 \cdot 12}{3} = 4 \cdot 4 = 16$.

41. A triangle has sides measuring 6 m, 8 m, and 12 m. When the angle bisector of the smallest angle is drawn, what is the length, in meters, of the smaller segment into which it divides the opposite side?

The smallest angle is opposite the smallest side, the 6. The angle bisector will divide that segment proportionally to the adjacent sides, so the smaller segment will be $\frac{8}{12+8} \cdot 6 = \frac{8}{20} \cdot 6 = \frac{2}{5} \cdot 6 = \frac{12}{5}$.

42. The diagram to the right shows a circle with intersecting secant and tangent line segments, with angles and arcs measured in degrees. What is the value of *k*?

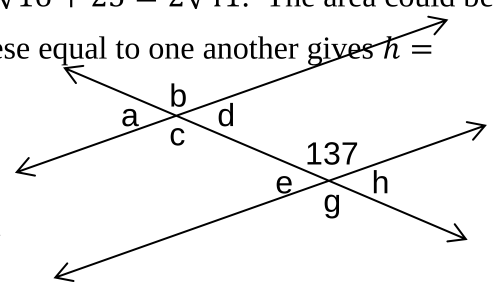


The unknown arcmeasure is $360 - 136 - 48 = 360 - 184 = 176°$, making $k = \frac{176-48}{2} = \frac{128}{2} = 64°$.

43. A right triangle has legs measuring 8 m and 10 m. What is the length, in meters, of the altitude to the hypotenuse?

The hypotenuse will be $\sqrt{8^2 + 10^2} = 2\sqrt{4^2 + 5^2} = 2\sqrt{16 + 25} = 2\sqrt{41}$. The area could be calculated as $\frac{8 \cdot 10}{2} = 40$, or $\frac{h \cdot 2\sqrt{41}}{2} = h\sqrt{41}$. Setting these equal to one another gives $h = \frac{40}{\sqrt{41}} = \frac{40\sqrt{41}}{41}$.

44. The diagram to the right shows two parallel lines intersected by a third line, with angles measured in degrees. What is the value of $a + d + h$?



h is supplementary to 137, so it is $180 - 137 = 43$. *a*, *d*, and *h* are all equal, so the answer is $3 \cdot 43 = 129$.

45. A llama is tethered to an outside corner of a square barn. If the rope is 9 m long and the barn has sides measuring 5 m each, what is the total area, in square meters, that the llama can graze?

The llama can graze three-quarters of a circle of radius 9, and two different quarter-circles of radius $9 - 5 = 4$, for an answer of $\frac{3}{4} \cdot \pi \cdot 9^2 + \frac{2}{4} \cdot 4^2 = \frac{243\pi + 32\pi}{4} = \frac{275\pi}{4}$.

46. Two circles with radii of 10 m have their centers 29 m apart. What is the length of a line segment between the two circles that is tangent to each circle and passes between them?

Draw the circles, the line segment between their centers, an internal tangent, and the radii to that tangent (making right angles). No draw a line segment congruent and parallel to the internal tangent from one of the centers to meet the extension of the radius from the other center. This creates a large right triangle with a hypotenuse of 29 and a "radius" leg measuring $10+10=20$. The "tangent" leg of this triangle must thus be $\sqrt{29^2-20^2}=\sqrt{841-400}=\sqrt{441}=21$.

47. How many diagonals can be drawn in a convex 14-gon?

Any one vertex can have $n-3$ diagonals drawn from it, because it cannot draw to itself or either neighbor. One might think this means there are $n(n-3)$ diagonals, but this would count each diagonal twice, once from the "start" and once from the "end", so our actual answer is $\frac{n(n-3)}{2}=\frac{14\cdot 11}{2}=7\cdot 11=77$.

48. What is the largest number of regions into which a plane can be divided by two lines and a triangle?

The triangle divides the plane into two regions. The first line can divide each of those two regions into two sub-regions, for a total of four. The second line can divide all four regions, for an answer of eight.

49. What is the measure, in degrees, of an angle which is complementary to a $16°$ angle?

Complementary angles sum to 90°, making our answer $90-16=74°$.

50. A regular polygon has vertices labeled in clockwise order from A to N. When a line is drawn through vertex H and the center of the polygon, what other vertex does it pass through?

A-N is 14 letters (M is 13), so we want the letter $14\div 2=7$ letters away from H, which is I-J-K-L-M-N-A.

51. A cube of white plastic is painted blue on all of its faces, then cut into 125 congruent cubes. How many of the smaller cubes have paint on exactly two faces?

The big cube was sliced into five slices in each dimension, so to have paint on exactly two faces, a smaller cube would have to have been on an edge of the big cube, but not on a vertex. There would be 5 smaller cubes along each edge of the big cube, which means $5-2=3$ that weren't vertices. There are 12 edges on a cube, so our answer is $3\cdot 12=36$.

52. Evaluate: $\boldsymbol{i(2-3i)+4i^5}$

$i(2-3i)+4i^5=2i+3+4i=6i+3$

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53. Given a point and a line, consider the set of all points that are twice as far from the point as they are from the line. What is the name of the conic section that these points are a part of?

The points that are *equidistant* from a point and line are a parabola. Those that are a particular amount closer to the point are ellipses, and those that are a particular amount farther are hyperbolae.

54. What is the area inside the locus of points satisfying $\frac{(x-1)^2}{2^2} + \frac{(y-3)^2}{4^2} = 1$?

This is an ellipse with a semi-major axis of 4 and a semi-minor axis of 2, for an area of $\pi \cdot 2 \cdot 4 = 8\pi$.

55. Evaluate: $\log_4 4096$

$4 = 2^2$ and $4096 = 2^{12} = (2^2)^6$, so the answer is 6.

56. If $b(c) = c^3 - 9$, evaluate $b^{-1}(334)$.

We want the value of *c* that makes $c^3 - 9 = 334$. This becomes $c^3 = 343$, with a solution of $c = 7$.

57. If $d(f) = 9 - \sqrt{87 - 6f}$ has a domain and range which are subsets of the real numbers, express the domain in interval notation.

Everything is in the domain unless it somehow breaks the function. Common things that can break are fractions (divide by 0), square roots (less than 0), and logarithms (less than 0). In this case, $87 - 6f \geq 0$, so $87 \geq 6f$, and thus $\frac{87}{6} = \frac{29}{2} \geq f$, which in interval notation is $\left(-\infty, \frac{29}{2}\right]$.

58. If g is directly proportional to h and $g = 24$ when $h = 36$, what is g when $h = 24$?

h was multiplied by $\frac{24}{36} = \frac{2}{3}$, so *g* will be as well, giving $24 \cdot \frac{2}{3} = 8 \cdot 2 = 16$.

59. What is the sum of the roots of $3j^3 - 2j^2 + j - 6 = 0$?

The sum of the roots of any polynomial is $-\frac{b}{a}$, which in this case is $-\frac{-2}{3} = \frac{2}{3}$.

60. What is the coefficient of the k^3 term when $(2k - 3)^5$ is expanded and like terms are combined?

The coefficient of each k^3 term will be $2^3(-3)^2 = 8 \cdot 9 = 72$, and there will be $5c3 = \frac{5 \cdot 4}{2} = 5 \cdot 2 = 10$ of them, for a total of 720.

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61. Evaluate: $\left(\frac{25}{9}\right)^{\frac{3}{2}}$

$$\left(\frac{25}{9}\right)^{\frac{3}{2}} = \left(\left(\frac{25}{9}\right)^{\frac{1}{2}}\right)^{3} = \left(\frac{5}{3}\right)^{3} = \frac{125}{27}$$

62. Express the base ten numeral 123_{10} as a base three numeral.

In base three, the digits represent $3^0 = 1$'s, $3^1 = 3$'s, $3^2 = 9$'s, $3^3 = 27$'s, $3^4 = 81$'s, etc. We'll need one 81, leaving $123 - 81 = 42$, then one 27, leaving $42 - 27 = 15$, then one 9, leaving $15 - 9 = 6$, then two 3's and no 1's, for an answer of 11120.

63. What is the prime factorization, in exponential form, of 594?

$594 = 2 \cdot 297 = 2 \cdot 3^2 \cdot 33 = 2 \cdot 3^3 \cdot 11$

64. How many positive integers are factors of 603?

$603 = 3 \cdot 201 = 3^2 \cdot 67^1$, so it has $(2 + 1)(1 + 1) = 3 \cdot 2 = 6$ factors.

65. What is the sum of the positive integer factors of 360?

$360 = 2^3 \cdot 3^2 \cdot 5$, so the sum of the factors will be $(1 + 2 + 4 + 8)(1 + 3 + 9)(1 + 5) = 15 \cdot 13 \cdot 6 = 195 \cdot 6 = 1170$.

66. What is the least common multiple of 45 and 80?

$45 = 3^2 \cdot 5$ and $80 = 2^4 \cdot 5$, so their LCM will be $2^4 \cdot 3^2 \cdot 5 = 80 \cdot 9 = 720$.

67. How many positive four-digit integers are composed of four different digits?

The first digit could be 1-9 (9 choices), the next could be 0-9 (10 choices, except we can't use whatever we used first, so 9 choices), the third could be 0-9 (minus two, so 8 choices), and the fourth could be 0-9 (minus three, so 7 choices). Thus, there are $9 \cdot 9 \cdot 8 \cdot 7 = 81 \cdot 56 = 4536$ such numbers.

68. What is the sum of the numbers that are multiples of five in the list 82, 1465, 63, 75, 780?

The multiples of 5 are 1465, 75, and 780, for an answer of $1465 + 75 + 780 = 1540 + 780 = 2320$.

69. What is the units digit of 987^{654}?

The units digit of an exponential expression depends only on the units digit of the base, so this problem has the same answer as 7^{654}. 7^1 ends in 7, $7^2 = 49$ ends with a 9, 7^3 ends with a 3 like $7 \cdot 9 = 63$, and 7^4 ends with a 1 like $7 \cdot 3 = 21$. After this, the cycle will repeat every four terms. 654 is 2 more than a multiple of 4, so it will end with a 9, like 7^2.

70. What is the seventh term of a geometric sequence with first term 6 and common ratio 2?

The 7th term is 6 ratios from the first term, for an answer of $6 \cdot 2^6 = 6 \cdot 64 = 384$.

71. What is the missing term of the sequence 2, 4, 6, 28, 18, 52, 54, __, 162, 100?

After trying several things including the sequence of differences, you may consider whether this is in fact two sequences that have been interspersed. Indeed, 2, 6, 18, 54, 162 is a geometric sequence, and 4, 28, 52, (76), 100 is an arithmetic sequence with common difference 24 giving an answer of 76.

72. What is the 37th term of an arithmetic sequence with first term 47 and common difference 17?

The 37th term will be 36 differences from the first term, for an answer of $47 + 36 \cdot 17 = 47 + 612 = 659$.

73. What is the sum of the 24 smallest positive integers?

Using the formula $\frac{n(n+1)}{2}$ gives $\frac{24 \cdot 25}{2} = 12 \cdot 25 = 300$.

74. What is the sum of the 13 smallest positive odd integers?

Using the formula n^2 gives $13^2 = 169$.

75. What is the sum of the 15 smallest positive perfect squares?

Using the formula $\frac{n(n+1)(2n+1)}{6}$ gives $\frac{15(15+1)(2 \cdot 15+1)}{6} = 5 \cdot 8 \cdot 31 = 40 \cdot 31 = 1240$.

76. What is the sum of the 8 smallest positive perfect cubes?

Using the formula $\left[\frac{n(n+1)}{2}\right]^2$ gives $\left[\frac{8(8+1)}{2}\right]^2 = (4 \cdot 9)^2 = 36^2 = 1296$.

77. When four fair coins are flipped, what is the probability that exactly two of them show tails?

Each coin can land in 2 ways, so there are a total of $2^4 = 16$ ways for the four coins to land. Of these, there are $4c2 = \frac{4 \cdot 3}{2} = 2 \cdot 3 = 6$ ways to have two tails, for a probability of $\frac{6}{16} = \frac{3}{8}$.

78. A trusted friend rolls two standard six-sided dice behind a screen and declares that she did not roll doubles. What is the probability she rolled a sum of 7?

She didn't get 11, 22, 33, 44, 55, or 66, so there are only 30 things she might have rolled. There are six ways to roll a sum of 7 (16, 25, 34, 43, 52, 61), for a probability of $\frac{6}{30} = \frac{1}{5}$.

79. The probability that it rains tomorrow is $\frac{1}{4}$, and the probability that I read a book tomorrow is $\frac{2}{3}$. If these events are independent, what is the probability that it rains but I do not read a book?

The probability I *don't* read a book is $1 - \frac{2}{3} = \frac{1}{3}$, for an answer of $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

80. In how many ways can the letters in the word "HORROR" be arranged?

There are $6! = 720$ ways to arrange six distinguishable items, but we have two identical O's that could be interchanged in $2! = 2$ ways without us noticing, and three R's that could be interchanged in $3! = 6$ ways, for an answer of $\frac{720}{2 \cdot 6} = \frac{720}{12} = 60$.

81. In how many ways can seven people sit relative to one another at a round table?

Where the first person sits is irrelevant. Once they've sat, there are six people who could be to their right, five for the next seat, etc., for an answer of $6! = 720$.

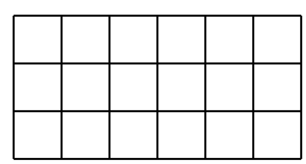
82. At my family reunion, 37 people liked hot dogs and 46 people liked hamburgers. If 13 people liked both and 31 liked neither, how many people were at the family reunion?

You can add the 31 who liked neither, the 46 who liked hamburgers, and the $37 - 13 = 24$ who like hot dogs but not hamburgers to get $31 + 46 + 24 = 31 + 70 = 101$.

83. I pick two random real numbers from 1 to 5 and plot a point on the Cartesian plane using the first number as the x-coordinate and the second as the y-coordinate. What is the probability that this point satisfies $y > 2x$?

Graphing the possible points, the total area they could lie in is $4 \cdot 4 = 16$, and the "good" area is a right triangle in the upper left measuring $\frac{1}{2} \cdot 3 \cdot \frac{3}{2} = \frac{9}{4}$, for a probability of $\frac{\frac{9}{4}}{16} = \frac{9}{64}$.

84. In the grid of unit squares to the right, how many paths of length 9 are there from the top left corner to the bottom right corner?



To travel between those corners, you must take 9 steps: 3 down and 6 over. There are $9c3 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = 3 \cdot 4 \cdot 7 = 12 \cdot 7 = 84$ ways to do that.

85. In how many ways can 8 identical candies be distributed among four children if fairness is not considered?

Throw three "change kids" rocks into the bag and start giving candy to the first kid. When you hit the first rock, switch to the second kid, etc. The rocks could come out in $\binom{8+3}{3} = \binom{11}{3} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2} = 11 \cdot 5 \cdot 3 = 11 \cdot 15 = 165$ ways.

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Solutions

86. What is the range of the data set {4, 71, 58, 60, 5, 13, 61, 35, 70}?

The range is the greatest element minus the least element, which in this case is $71 - 4 = 67$.

87. Set C is {70, 5, 93, 57, 3} and Set D is {57, 90, 71, 3, 683}. Write the set $\boldsymbol{C \cap D}$.

The intersection is the elements that are in both sets, which is $\{57,3\}$.

88. Set E is {5, 6, 7, 93} and Set F is {93, 5, 7}. How many subsets of E are supersets of F?

To be a superset of F, you must contain 93, 5, and 7. To be a subset of E, you can *only* contain 5, 6, 7, and 93. Thus, the sets we're interested in must have 93, 5, and 7, and *may* include 6. There are two such sets; one contains the 6, and the other doesn't.

89. If $\boldsymbol{G = \begin{bmatrix} 9 & 4 \\ 4 & 2 \end{bmatrix}}$, what is $\boldsymbol{G^{-1}}$?

The inverse of a matrix is the transposed matrix of cofactors, divided by the determinant. In this case the determinant is $\begin{vmatrix} 9 & 4 \\ 4 & 2 \end{vmatrix} = 9 \cdot 2 - 4 \cdot 4 = 18 - 16 = 2$, and the transposed matrix of cofactors is $\begin{bmatrix} 2 & -4 \\ -4 & 9 \end{bmatrix}$, for an answer of $\begin{bmatrix} 1 & -2 \\ -2 & \frac{9}{2} \end{bmatrix}$.

90. In the matrix $\boldsymbol{\begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}}$, what is the cofactor of the 6?

The cofactor of the 6 is $(-1)\begin{vmatrix} 1 & 2 \\ 7 & -8 \end{vmatrix} = (-1)(1(-8) - 7 \cdot 2) = (-1)(-8 - 14) = 22$.

91. In the cross-number puzzle to the right, each of U, V, W, and X is a positive one-digit integer collectively satisfying four equations: two horizontal, two vertical. What is the product of U, V, W, and X?

U	x	V	=	8
-		+		
W	+	X	=	8
=		=		
3		9		

U & V might be 1 & 8 or 2 & 4 in either order. U cannot be 1 or 2 in the vertical subtraction, so U, V, & W might be 4, 2, & 1 or 8, 1, & 5. In the former case, X could be 7 by both equations. In the latter case X could be 8 by the vertical equation but 3 by the horizontal equation, which won't work. Thus the answer is $4 \times 2 \times 1 \times 7 = 56$.

92. Evaluate: $\boldsymbol{\frac{1}{4} \cdot \frac{2}{5} \cdot \frac{3}{6} \cdots \frac{10}{13} \cdot \frac{11}{14} \cdot \frac{12}{15}}$

There will be a lot of canceling, starting with the first 4 in the denominator and the 4 in the numerator of the first missing term. After the cancel-fest, we'll be left with $\frac{1 \cdot 2 \cdot 3}{13 \cdot 14 \cdot 15}$, which cancels to give $\frac{1}{13 \cdot 7 \cdot 5} = \frac{1}{455}$.

93. In a right triangle with legs measuring 3 m and 5 m, what is the tangent of the smallest angle?

The smallest angle is opposite the smallest side, which is the 3 (regardless of the length of the hypotenuse), so the tangent will be $\frac{3}{5}$.

94. Express $216°$ in radians.

$216° \cdot \frac{\pi}{180°} = \pi \cdot \frac{216}{180} = \pi \cdot \frac{36}{30} = \frac{6\pi}{5}$

95. What is the period of $y = 7\cos(6x - 5) + 4$?

The period of the cosine function is normally 2π, so the period of this one will be $\frac{2\pi}{6} = \frac{\pi}{3}$.

96. Evaluate the function: $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

We're looking for the angle whose sine is $-\frac{\sqrt{3}}{2}$, between $\pm\frac{\pi}{2}$. This will be $-\frac{\pi}{3}$.

97. If $m(t) = 3t^4$, evaluate $m'(-2)$.

$m'(t) = 12t^3$, so $m'(-2) = 12(-2)^3 = 12(-8) = -96$.

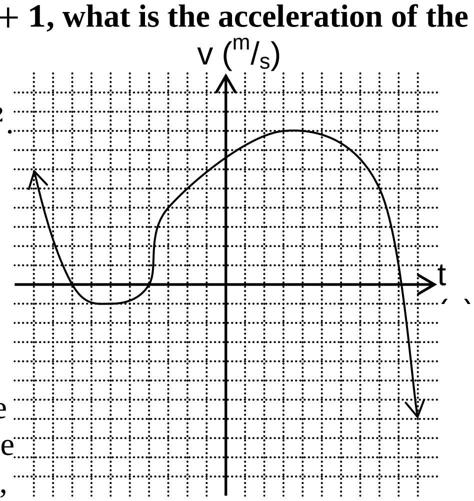
98. Evaluate: $\int_1^2 (n(n^2 - 3)^4)dn$

This becomes $\frac{1}{2} \cdot \frac{1}{5}(n^2 - 3)^5\Big|_1^2 = \frac{1}{10}(1^5 - (-2)^5) = \frac{1}{10}(1 + 32) = \frac{33}{10}$.

99. If the velocity of a particle on a line is $v(t) = 3e^{2t} + 1$, what is the acceleration of the particle when $t = -1$?

Taking the derivative, $a(t) = 6e^{2t}$, so $a(-1) = 6e^{-2}$.

100. The velocity of a particle moving along a line is graphed to the right on a unit grid. At what value(s) of *t* does the particle change direction?



Positive values of *v* indicate that the particle is traveling to the right, while negative values indicate that the particle is traveling to the left, so places where it switches sign (by passing through 0) indicate that the particle has changed direction. This occurs at $t = -8$, -4, and 9.