

2015 Fall Startup Event
Thursday, September 24th, 2015

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

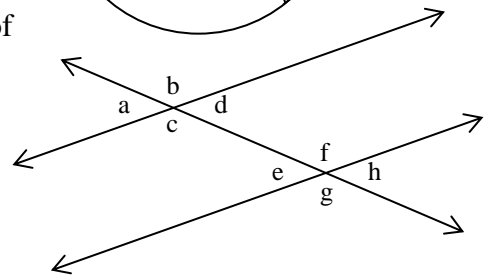
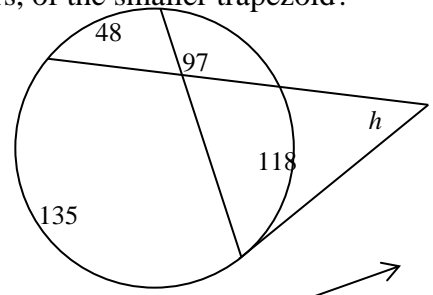
1. Evaluate: $8239 \div 7$
2. What is the remainder when 836 is divided by 39?
3. Evaluate **as a decimal**: $1.2 + 12.34 + 123.4$
4. Evaluate **as a fraction**: $\frac{4}{5} - \frac{1}{3}$
5. Evaluate **as a mixed number**: $\frac{5}{6} \times 1\frac{3}{5}$
6. Evaluate: $\frac{9!}{6!}$
7. Evaluate: $8 \div 4 + (5 \times 6 - 9 \div 3) - 8 + 9$
8. Express in simplest radical form: $\sqrt{325}$
9. Express in simplest radical form: $\sqrt[4]{240}$
10. Evaluate: 119×121
11. Evaluate: $3^3 + 4^4$
12. When the magic number is reduced by 7 and this result is divided by 3, the final result is 24. What is the magic number?
13. What value(s) of b satisfy $9b + 1 = 73$?
14. What value(s) of z satisfy $2z - 8 = 7z - 93$?
15. What is the solution, in the form (d, f) , of the system of equations $d + 2f = 0$ and $d - f = 6$?
16. If three chickens can collectively lay four eggs in five days, how many eggs could eighteen chickens lay ten days?
17. If 20 liters of a 20% acid solution is combined with 80 liters of an 80% acid solution, what percentage of the resulting solution will be acid?

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18. Nathalie drove for four hours at a speed of 35 mph, then for six hours at a speed of 60 mph. What was her average speed for the entire trip, in miles per hour?
19. Two friends start at the same point and run the same direction around a quarter-mile track. If one of them runs at five miles per hour and the other runs at a speed of nine miles per hour, how many times will they be at the same point on the track (including their initial co-location) if they run for two hours?
20. The sum of two numbers is 89 and the difference between those two numbers is 37. What is the smaller of the two numbers?
21. In which quadrant of the Cartesian Plane does the point $(73, -8)$ lie?
22. What is the equation, in slope-intercept ($y = mx + b$) form, of the line through the points $(6, -5)$ and $(4, -1)$?
23. What is the slope of a line perpendicular to the line $2x - 3y = 4$ and passing through the point $(-2, 58)$?
24. What is the distance between the points $(-5, -8)$ and $(-9, -2)$?
25. What are the coordinates, in the form (x, y) , of the point of intersection of the lines $y = 3x - 7$ and $2x + y = 8$?
26. When Ms. Miz writes an equation of the form $f^2 + bf + c = 0$ on the board, Kid miscopies the value of b and gets roots of 9 and -2, while Kitty miscopies the value of c and gets roots of $\frac{-3 \pm i}{2}$. What were the roots of Ms. Miz's original equation?
27. A square picture with an area of 144 cm^2 is glued in the center of a square piece of paper that extends 2 cm beyond the picture on all sides. What is the area, in square centimeters, of the paper that can be seen around the edges of the picture?
28. 2 goats can be exchanged for 3 hyenas and 4 hyenas can be exchanged for one iguana. For how many goats could 30 iguanas be exchanged?
29. I have nine coins in my pocket worth a total of 99 cents. How many dimes do I have? Assume only pennies, nickels, dimes, and quarters could be in my pocket.
30. What value(s) of y satisfy $\frac{y+2}{y-1} = \frac{y-1}{y-9}$?
31. If $h(x) = 4 + 7x$, evaluate $h(5)$.
32. A right triangle has legs measuring 7 m and 9 m. What is the length, in meters, of its hypotenuse?
33. A right triangle has an angle measuring 45° and a hypotenuse measuring 8 m. What is the area, in square meters, of the triangle?

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34. What is the most specific name that applies to **every** triangle with at least two congruent sides?
35. What is the most specific name that applies to **every** quadrilateral with four congruent angles?
36. What is the name of the point where the perpendicular bisectors of the sides of a triangle intersect?
37. What is the area, in square meters, of a circle with a radius measuring 6 m?
38. What is the perimeter, in meters, of a regular octagon with sides measuring 5 m?
39. What is the volume of a right rectangular pyramid with base edges measuring 4 m and 3 m, and a height of 2 m?
40. An equilateral triangle is inscribed in a circle. If the sides of the triangle measure 6 m, what is the area, in square meters, of the circle?
41. Four interior angles in a pentagon measure 90° , 80° , 70° , and 150° . What is the measure of the fifth interior angle in the pentagon?
42. Two similar trapezoids have bases measuring 3 m & 5 m and 9 m & 15 m. If the area of the larger trapezoid is 981 m^2 , what is the area, in square meters, of the smaller trapezoid?
43. In the figure to the right, some angles and arcs are labeled in degrees. What is the value of h ?
44. In a triangle with sides measuring 18 m, 14 m, and 10 m, a line is drawn bisecting the largest angle and passing through the opposite side, dividing it into two line segments. What is the length, in meters, of the longer of these two segments?
45. In the figure to the right, parallel lines are intersected by a third line. If $m\angle g = 132^\circ$, what is the sum, in degrees, of the measures of angles a , c , and e ?
46. What is the largest number of regions into which a plane can be divided by two lines and two ellipses?
47. What is the measure, in degrees, of the complement to the supplement of 111° ?
48. The vertices of a regular polygon are labeled in clockwise order from A to R. If a line is drawn through vertex F bisecting the polygon, what other vertex will the line pass through?



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49. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 11:40 PM?
50. How many real numbers satisfy $2c^2 + 16c = -32$?
51. Evaluate in terms of $i = \sqrt{-1}$: $(4i - 3)^2$
52. What is the name for a locus of points that are half as far from a given point as they are from a given line?
53. In how many points do the graphs of $x = (y - 1)^2 + 7$ and $\frac{(x+2)^2}{400} + \frac{(y-3)^2}{100} = 1$ intersect?
54. What is the smallest integer value of w for which the value of $3 \times 2^w - 1$ is greater than 1000?
55. Katium has a half-life of 3 hours. If you have a 1 kg sample of Katium when you go to bed at 10 PM, how many grams will remain when you wake up at 7 AM?
56. What is the sum of the roots of $2v^2 + 5v - 11 = 0$?
57. Evaluate: $32^{-\frac{3}{5}}$
58. What is the smallest prime number greater than 90?
59. Express the sum of the base 7 numbers 3461_7 and 5645_7 as a base 7 number.
60. Express the base 3 number 12120_3 as a base 9 number.
61. What is the prime factorization, in exponential form, of 676?
62. How many positive integers are factors of 891?
63. How many positive integers are factors of 234 and multiples of 6?
64. How many positive three-digit integers do not have 3 or 7 for any of their digits?
65. Which of the listed numbers are multiples of three?
5, 92, 789, 6783, 92345
66. What is the units digit of 37^{73} ?
67. What is the sixth term of a geometric sequence with first term 13 and common ratio 2?
68. What is the next term of a sequence that begins with 7, 11, 17, 25, 35, and 47?
69. What is the next term of a harmonic sequence that begins with 4, 3, $\frac{12}{5}$, and 2?

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70. The first term of a sequence is $u_1 = 3$, and subsequent terms are $u_n = 2u_{n-1} + 5$. What is the sixth term of this sequence?
71. Evaluate: $\sum_2^{100} \left(\frac{1}{n-1} - \frac{1}{n} \right)$
72. What is the sum of the terms of an infinite geometric sequence with first term 24 and common ratio $\frac{1}{4}$?
73. What is the sum of the 15 smallest positive odd numbers?
74. What is the sum of the even numbers between 21 and 59?
75. One marble is drawn from a bag containing 3 red marbles, 5 white marbles, and 7 blue marbles. What is the probability that the marble drawn is white?
76. One card is drawn from a standard 52-card deck. What is the probability that the card drawn is either a king or a heart (or both)?
77. When four fair coins are flipped, what is the probability that exactly one of them displays heads?
78. When two standard six-sided dice are rolled, what is the probability that the sum of the numbers shown is four?
79. Evaluate: $\binom{7}{3}$
80. When five people sit at a round table, how many relative arrangements are possible?
81. Ralph and Thea plan to meet at the game shop sometime between 4 PM and 5 PM. If each shows up at a random time in that interval, waits up to 30 minutes for the other, and then leaves if the other one hasn't shown up, what is the probability that they will meet as planned?
82. On the first day of school, Mrs. Tessandore has five identical pencils to distribute among her three students. If fairness doesn't matter to her, how many different distributions of pencils are possible?
83. What is the median of the data set {589, 78, 9345, 7, 894}?
84. What is the population standard deviation of the data set {0, 4}?
85. Set L is the set of positive one-digit composite integers and Set S is the set of positive one-digit integers. How many supersets of Set L are subsets of Set S?
86. If $\mathbf{m} = \langle 4, -1 \rangle$ and $\mathbf{r} = \langle 1, 3 \rangle$, evaluate $2\mathbf{r} - 3\mathbf{m}$.
87. What is the area of a triangle with vertices at the points (2,5), (-3,2), and (-1,5)?

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88. In the cryptarithm below, each instance of a given letter represents the same digit (0-9), and different letters represent different digits. What is the largest possible value of the four-digit

$$\begin{array}{r} AB \\ \text{positive integer } ABCD? \quad \frac{-C}{D} \end{array}$$

$$\boxed{A} \times \boxed{B} = \boxed{12}$$

89. In the cross-number puzzle to the right, A , B , C , and D are different digits that together satisfy the four equations (two across and two down). What is the product of A , B , C , and D ?

$$\begin{array}{r} \times \quad + \\ \boxed{C} + \boxed{D} = \boxed{12} \end{array}$$

90. Evaluate: $\frac{4}{4 - \frac{4}{4 - \frac{4}{4 - \frac{4}{4}}}}$

$$\begin{array}{r} = \quad = \\ \boxed{18} \quad \boxed{9} \end{array}$$

91. If $\cos p = \frac{4}{5}$ and $0 < p < \pi$, evaluate $\sin p$.

92. What is the area, in square meters, of a triangle with sides measuring 3 m, 5 m, and 6 m?

93. Evaluate: $\cot \frac{5\pi}{3}$

94. What are the rectangular coordinates, in the form (x, y, z) , of the cylindrical coordinates $(6, \frac{2\pi}{3}, -1)$?

95. What is the remainder when $b^5 - 4b^3 + 2b - 9$ is divided by $b + 2$?

96. Evaluate: $\lim_{c \rightarrow 3^-} [c]$

97. Evaluate: $\lim_{w \rightarrow \infty} \frac{5w^2 - 4w + 3}{2w^2 + 9}$

98. If $d(v) = 2(\ln v)^3$, evaluate $d'(e)$.

99. If $u(f) = 3\pi^2 + 1$, evaluate $u'(4)$.

100. What is the area of the region bounded by $y = x^2$ and $y = x^3$?