2015 Fall Startup Event
Solutions

1. Evaluate: $8239 \div 7$

The standard division algorithm gives $1000 + 100 + 70 + 7 = 1177$.

2. What is the remainder when 836 is divided by 39?

Again, the standard algorithm gives $20 + 1 = 21$ with a remainder of $56 - 39 = 17$.

3. Evaluate as a decimal: $1.2 + 12.34 + 123.4$

Aligning the decimal points gives 136.94.

4. Evaluate as a fraction: $\frac{4}{5} - \frac{1}{3}$

$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{12-5}{15} = \frac{7}{15}$

5. Evaluate as a mixed number: $\frac{5}{6} \times 1\frac{3}{5}$

$\frac{5}{6} \times 1\frac{3}{5} = \frac{5}{6} \times \frac{8}{5} = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$

6. Evaluate: $\frac{9!}{6!}$

$\frac{9!}{6!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1} = 9 \times 8 \times 7 = 8^3 - 8 = 512 - 8 = 504$

7. Evaluate: $8 \div 4 + (5 \times 6 - 9 \div 3) - 8 + 9$

$8 \div 4 + (5 \times 6 - 9 \div 3) - 8 + 9 = 2 + (30 - 3) + 1 = 2 + 27 + 1 = 30$

8. Express in simplest radical form: $\sqrt{325}$

$\sqrt{325} = \sqrt{25 \times 13} = 5\sqrt{13}$

9. Express in simplest radical form: $\sqrt[4]{240}$

$\sqrt[4]{240} = \sqrt[4]{16 \times 15} = 2\sqrt[4]{15}$

10. Evaluate: 119×121

$119 \times 121 = (120 - 1)(120 + 1) = 120^2 - 1 = 14400 - 1 = 14399$

11. Evaluate: $3^3 + 4^4$

$3^3 + 4^4 = 27 + 256 = 283$

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12. When the magic number is reduced by 7 and this result is divided by 3, the final result is 24. What is the magic number?

Working backwards, the intermediate result was $24 \times 3 = 72$ and the magic number was $72 + 7 = 79$.

13. What value(s) of $\boldsymbol{b}$ satisfy $\mathbf{9}\boldsymbol{b} + \mathbf{1} = \mathbf{73}$?

$9b + 1 = 73$ becomes $9b = 72$, giving $b = 8$.

14. What value(s) of $\boldsymbol{z}$ satisfy $\mathbf{2}\boldsymbol{z} - \mathbf{8} = \mathbf{7}\boldsymbol{z} - \mathbf{93}$?

$2z - 8 = 7z - 93$ becomes $85 = 5z$, giving $17 = z$.

15. What is the solution, in the form $(\boldsymbol{d}, \boldsymbol{f})$, of the system of equations $\boldsymbol{d} + \mathbf{2}\boldsymbol{f} = \mathbf{0}$ and $\boldsymbol{d} - \boldsymbol{f} = \mathbf{6}$?

Subtracting the two equations gives $3f = -6$, which yields $f = -2$, so that $d = 4$ by either equation.

16. If three chickens can collectively lay four eggs in five days, how many eggs could eighteen chickens lay ten days?

We're multiplying the number of chickens by 6, so we'd expect to multiply the number of eggs by 6 as well. We're also multiplying the number of days by 2, so we should multiply the number of eggs by 2 also. Thus, our answer is $4 \times 6 \times 2 = 48$.

17. If 20 liters of a 20% acid solution is combined with 80 liters of an 80% acid solution, what percentage of the resulting solution will be acid?

The final solution will be $80 + 20 = 100$ liters, $20 \times .2 + 80 \times .8 = 4 + 64 = 68$ liters of which is acid, for an answer of $\frac{68}{100} = 68\%$.

18. Nathalie drove for four hours at a speed of 35 mph, then for six hours at a speed of 60 mph. What was her average speed for the entire trip, in miles per hour?

She went a total of $4 \times 35 + 6 \times 60 = 140 + 360 = 500$ miles in $4 + 6 = 10$ hours, for an average speed of $\frac{500}{10} = 50$.

19. Two friends start at the same point and run the same direction around a quarter-mile track. If one of them runs at five miles per hour and the other runs at a speed of nine miles per hour, how many times will they be at the same point on the track (including their initial co-location) if they run for two hours?

The faster person goes $9 - 5 = 4$ mph faster than the slower person, so in two hours she goes $2 \times 4 = 8$ more miles, which is $\frac{8}{\frac{1}{4}} = 32$ more laps. Counting the original co-location gives an answer of $32 + 1 = 33$.

20. The sum of two numbers is 89 and the difference between those two numbers is 37. What is the smaller of the two numbers?

The difference in the two given numbers is $89 - 37 = 52$, which represents the difference between adding the smaller number and subtracting the smaller number, so the smaller number must be $\frac{52}{2} = 26$.

21. In which quadrant of the Cartesian Plane does the point $(73, -8)$ lie?

This point lies to the right and below the origin, which is the fourth quadrant, typically referred to as IV.

22. What is the equation, in slope-intercept $(y = mx + b)$ form, of the line through the points $(6, -5)$ and $(4, -1)$?

The slope is $\frac{-5-(-1)}{6-4} = \frac{-4}{2} = -2$, so that the equation is $y = -2x + b$. Substituting the first point gives $-5 = -2 \times 6 + b = -12 + b$ so that $7 = b$ for an answer of $y = -2x + 7$.

23. What is the slope of a line perpendicular to the line $2x - 3y = 4$ and passing through the point $(-2, 58)$?

The point is a red herring; all we need to know is that the slope of the given line is $m = -\frac{a}{b} = -\frac{2}{-3} = \frac{2}{3}$. The slope of all perpendicular lines will be the negative reciprocal, which is $-\frac{3}{2}$.

24. What is the distance between the points $(-5, -8)$ and $(-9, -2)$?

The two points are $-5 - (-9) = 4$ apart in the x direction and $-2 - (-8) = 6$ apart in the y direction. Their distance will be the hypotenuse of a right triangle with legs of 4 and 6, which is twice the hypotenuse of a right triangle with legs of 2 and 3, for an answer of $2\sqrt{2^2 + 3^2} = 2\sqrt{4 + 9} = 2\sqrt{13}$.

25. What are the coordinates, in the form (x, y), of the point of intersection of the lines $y = 3x - 7$ and $2x + y = 8$?

Substituting the former into the latter gives $5x - 7 = 8$, so that $5x = 15$ and $x = 3$, giving $y = 3 \times 3 - 7 = 9 - 7 = 2$.

26. When Ms. Miz writes an equation of the form $f^2 + bf + c = 0$ on the board, Kid miscopies the value of b and gets roots of 9 and -2, while Kitty miscopies the value of c and gets roots of $\frac{-3 \pm i}{2}$. What were the roots of Ms. Miz's original equation?

Kid must have had the correct value of $c = 9(-2) = -18$, which Kitty's must have had the right $b = -2\left(-\frac{3}{2}\right) = 3$, so that the original equation was $f^2 + 3f - 18 = 0$. This factors to $(f + 6)(f - 3) = 0$ with roots of -6 and 3.

27. A square picture with an area of 144 cm^2 is glued in the center of a square piece of paper that extends 2 cm beyond the picture on all sides. What is the area, in square centimeters, of the paper that can be seen around the edges of the picture?

The sides of the picture are $\sqrt{144} = 12$ cm, so the sides of the paper are $12 + 2 + 2 = 16$. The area of the paper is thus $16^2 = 256$ cm^2, but we can't see 144 cm^2 of it, so the answer is $256 - 144 = 112$ cm^2.

28. 2 goats can be exchanged for 3 hyenas and 4 hyenas can be exchanged for one iguana. For how many goats could 30 iguanas be exchanged?

30 iguanas could be exchanged for $4 \times 30 = 120$ hyenas. This is $\frac{120}{3} = 40$ groups of three, which could thus be exchanged for $40 \times 2 = 80$ goats.

29. I have nine coins in my pocket worth a total of 99 cents. How many dimes do I have? Assume only pennies, nickels, dimes, and quarters could be in my pocket.

I must have 4 pennies, leaving 95 cents for the other five coins. I must have three quarters, which leaves 20 cents for the other two coins, making them both dimes, for an answer of 2.

30. What value(s) of y satisfy $\frac{y+2}{y-1} = \frac{y-1}{y-9}$?

Cross-multiplying gives $y^2 - 7y - 18 = y^2 - 2y + 1$, which becomes $-19 = 5y$, for an answer of $-\frac{19}{5}$.

31. If $h(x) = 4 + 7x$, evaluate $h(5)$.

$h(5) = 4 + 7 \times 5 = 4 + 35 = 39$

32. A right triangle has legs measuring 7 m and 9 m. What is the length, in meters, of its hypotenuse?

The Pythagorean Theorem gives $\sqrt{7^2+9^2}=\sqrt{49+81}=\sqrt{130}$.

33. A right triangle has an angle measuring $45°$ and a hypotenuse measuring 8 m. What is the area, in square meters, of the triangle?

A leg of the triangle is $\frac{8}{\sqrt{2}}$, so that its area is $\frac{1}{2}\left(\frac{8}{\sqrt{2}}\right)^2=\frac{1}{2}\left(\frac{64}{2}\right)=16$.

34. What is the most specific name that applies to every triangle with at least two congruent sides?

You just need to have memorized that these are **isosceles** triangles.

35. What is the most specific name that applies to every quadrilateral with four congruent angles?

Each angle will be $90°$, so that these shapes are **rectangles**.

36. What is the name of the point where the perpendicular bisectors of the sides of a triangle intersect?

The perpendicular bisectors are equidistant from the endpoints of the sides of the triangle, so the point where all three meet will be equidistant from all three vertices, which makes this point the center of the circumscribed circle, which is the **circumcenter**.

37. What is the area, in square meters, of a circle with a radius measuring 6 m?

$A=\pi r^2=36\pi$

38. What is the perimeter, in meters, of a regular octagon with sides measuring 5 m?

There are 8 sides, each measuring 5 m, for an answer of $8\times 5=40$.

39. What is the volume of a right rectangular pyramid with base edges measuring 4 m and 3 m, and a height of 2 m?

The volume will be $\frac{1}{3}Bh=\frac{1}{3}\times 4\times 3\times 2=4\times 2=8$.

40. An equilateral triangle is inscribed in a circle. If the sides of the triangle measure 6 m, what is the area, in square meters, of the circle?

Drawing all the altitudes in the triangle divides it into six congruent 30-60-90 triangles, each of which has a long leg of 3, a short leg of $\frac{3}{\sqrt{3}}=\sqrt{3}$ and a hypotenuse of $2\sqrt{3}$. The radius of the circle is also $2\sqrt{3}$, so that $A=\pi r^2=\pi\times\left(2\sqrt{3}\right)^2=\pi\times 4\times 3=12\pi$.

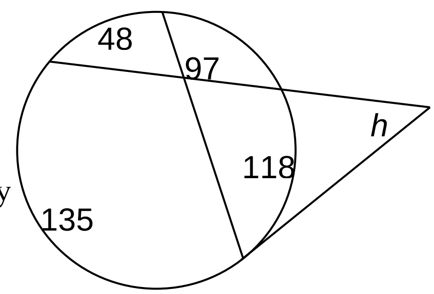
41. Four interior angles in a pentagon measure $90°$, $80°$, $70°$, and $150°$. What is the measure of the fifth interior angle in the pentagon?

The interior angles of a pentagon add up to $(5-2)\times 180° = 3 \times 180° = 540°$, and these angles add up to $90° + 80° + 70° + 150° = 390°$, so that the remaining angle must be $540° - 390° = 150°$.

42. Two similar trapezoids have bases measuring 3 m & 5 m and 9 m & 15 m. If the area of the larger trapezoid is 981 m^2, what is the area, in square meters, of the smaller trapezoid?

Every distance in the larger trapezoid must be $\frac{9}{3} = \frac{15}{5} = 3$ times the corresponding distance in the smaller trapezoid, so that the area of the larger trapezoid will be $3^2 = 9$ times the area of the smaller trapezoid, for an answer of $\frac{981}{9} = 109$.

43. In the figure to the right, some angles and arcs are labeled in degrees. What is the value of h?

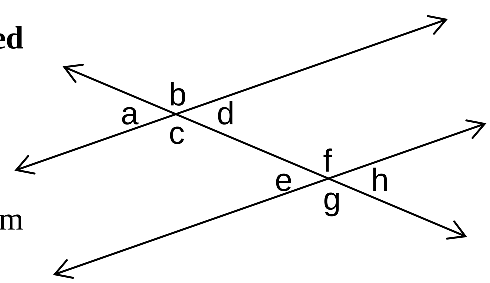


Although it's all consistent, only 135 and 118 are necessary to determine $h = \frac{135-118}{2} = \frac{17}{2}$.

44. In a triangle with sides measuring 18 m, 14 m, and 10 m, a line is drawn bisecting the largest angle and passing through the opposite side, dividing it into two line segments. What is the length, in meters, of the longer of these two segments?

Angle bisectors divide the opposite side proportionally to the lengths of the adjacent sides, so the longer segment will be $\frac{14}{10+14} \times 18 = \frac{14}{24} \times 18 = \frac{7}{2} \times 3 = \frac{21}{2}$.

45. In the figure to the right, parallel lines are intersected by a third line. If $m\angle g = 132°$, what is the sum, in degrees, of the measures of angles a, c, and e?



$a + c = 180°$, regardless of anything else, and $e = 180° - g = 180° - 132° = 48°$, so that the desired sum is $180° + 48° = 228°$.

46. What is the largest number of regions into which a plane can be divided by two lines and two ellipses?

The two ellipses could cross one another to create six regions (including the exterior). The first line could cross four of these regions to create four new regions for a subtotal of ten. Note that although this line crosses the exterior region twice, it only creates one new region there. Finally, the second line could cross six regions, creating six new ones for a total of 16.

47. What is the measure, in degrees, of the complement to the supplement of $111°$?

The supplement is $180° - 111° = 69°$, and its complement is $90° - 69° = 21°$.

48. The vertices of a regular polygon are labeled in clockwise order from A to R. If a line is drawn through vertex F bisecting the polygon, what other vertex will the line pass through?

R is the 18th letter, so there are 18 vertices. The dividing line should connect vertices that are 9 apart. G, H, I, J, K, L, M, N, O…

49. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 11:40 PM?

The numbers on a clock are $\frac{360°}{12} = 30°$ apart. At 11:40, the minute hand it pointing to the 8, and the hour hand is $\frac{40}{60} = \frac{2}{3}$ of the way from 11 to 12. From 8 to 11 is $3 \times 30° = 90°$, and from 11 to the hour hand is $\frac{2}{3} \times 30° = 20°$, for an answer of $90° + 20° = 110°$.

50. How many real numbers satisfy $2c^2 + 16c = -32$?

$2c^2 + 16c = -32$ becomes $2c^2 + 16c + 32 = 0$, then $c^2 + 8c + 16 = 0$, which factors to $(c + 4)^2 = 0$, so that there is only one value for c (-4).

51. Evaluate in terms of $i = \sqrt{-1}$: $(4i - 3)^2$

$(4i - 3)^2 = 16i^2 - 24i + 9 = -16 - 24i + 9 = -7 - 24i$

52. What is the name for a locus of points that are half as far from a given point as they are from a given line?

If the distances are equal, you have a parabola. If the point distance is greater, you have a hyperbola. If the point distance is less, you have an **ellipse**.

53. In how many points do the graphs of $x = (y - 1)^2 + 7$ and $\frac{(x+2)^2}{400} + \frac{(y-3)^2}{100} = 1$ intersect?

The first graph is a parabola heading to the right from (7,1), while the second is an ellipse centered at $(-2,3)$ with a width of 40 and a height of 20. The parabola starts inside the ellipse and thus intersects it twice.

54. What is the smallest integer value of w for which the value of $3 \times 2^w - 1$ is greater than 1000?

$3 \times 2^w - 1 > 1000$ becomes $3 \times 2^w > 1001$, then $2^w \sim > 333$, so that the smallest value of w will be 9, because $2^9 = 512$.

55. Katium has a half-life of 3 hours. If you have a 1 kg sample of Katium when you go to bed at 10 PM, how many grams will remain when you wake up at 7 AM?

From 10 PM to 7 AM is $7 - 10 + 12 = 9$ hours, which is $9 \div 3 = 3$ half-lives, so the remaining sample will be $1000 \times \left(\frac{1}{2}\right)^3 = \frac{1000}{8} = 125$.

56. What is the sum of the roots of $2v^2 + 5v - 11 = 0$?

The sum of the roots of a quadratic is $-\frac{b}{a} = -\frac{5}{2}$.

57. Evaluate: $32^{-\frac{3}{5}}$

$32^{-\frac{3}{5}} = \left(\frac{1}{32}\right)^{\frac{3}{5}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

58. What is the smallest prime number greater than 90?

$91 = 7 \times 13$, $93 = 3 \times 31$, $95 = 5 \times 19$, and 97 is prime.

59. Express the sum of the base 7 numbers 3461_7 and 5645_7 as a base 7 number.

You don't need to convert to base 10 to add, subtract, or multiply within another base, just use the standard algorithm with adjusted "carrying" or "regrouping". In this case, starting on the right, $1 + 5 = 6$, $6 + 4 = 10$ (3 carry 1), $4 + 6 + 1 = 11$ (4 carry 1), and $3 + 5 + 1 = 9$ (2 carry 1), for an answer of 12436_7.

60. Express the base 3 number 12120_3 as a base 9 number.

Because $9 = 3^2$, every two digits of a base 3 number can convert directly to a digit of a base 9 number. Starting on the right, $20_3 = 6_9$, $21_3 = 7_9$, and $1_3 = 1_9$, for an answer of 176_9.

61. What is the prime factorization, in exponential form, of 676?

$676 = 26^2 = (2 \times 13)^2 = 2^2 \times 13^2$

62. How many positive integers are factors of 891?

$891 = 9 \times 99 = 9 \times 9 \times 11 = 3^4 \times 11^1$, so a factor can have from zero to four 3's (5 choices) in *its* prime factorization, and can have from zero to one 11 (2 choices), for a total of $5 \times 2 = 10$ factors.

63. How many positive integers are factors of 234 and multiples of 6?

$6 = 2^1 \times 3^1$ and $234 = 2 \times 117 = 2 \times 3^2 \times 19$, so the numbers in question must have exactly one 2 in their prime factorization, from one to two 3's (2 choices), and from zero to one 19 (2 choices), for a total of $1 \times 2 \times 2 = 4$ numbers.

64. How many positive three-digit integers do not have 3 or 7 for any of their digits?

The hundreds digit could be one of 7 digits, and each of the other two digits could be one of 8 digits (zero is okay there), for an answer of $7 \times 8 \times 8 = 7 \times 64 = 448$.

65. Which of the listed numbers are multiples of three?
5, 92, 789, 6783, 92345

If the digits add up to a multiple of 3, then the number is a multiple of 3. You can ignore digits that are multiples of 3, or groups of digits that add up to a multiple of 3. 5 is not a multiple of 3, 92 is not, **789** is, **6783** is, and 92345 is not.

66. What is the units digit of 37^{73}?

The units digits of powers of 7 repeat every four terms, 7, 9, 3, 1, 7, 9, 3, 1, … so 7^{72} will end in a 1, so 37^{73} will end in a 7.

67. What is the sixth term of a geometric sequence with first term 13 and common ratio 2?

The sixth term is five ratios from the first term, so will be $13 \times 2^5 = 13 \times 32 = 320 + 96 = 416$.

68. What is the next term of a sequence that begins with 7, 11, 17, 25, 35, and 47?

The differences are 4, 6, 8, 10, and 12, so the next difference will be 14, for an answer of $47 + 14 = 61$.

69. What is the next term of a harmonic sequence that begins with 4, 3, $\frac{12}{5}$, and 2?

This is $\frac{12}{3}, \frac{12}{4}, \frac{12}{5}, \frac{12}{6}$, so the next term will be $\frac{12}{7}$.

70. The first term of a sequence is $u_1 = 3$, and subsequent terms are $u_n = 2u_{n-1} + 5$. What is the sixth term of this sequence?

$u_2 = 2u_1 + 5 = 2 \times 3 + 5 = 6 + 5 = 11$, $u_3 = 2 \times 11 + 5 = 22 + 5 = 27$, $u_4 = 2 \times 27 + 5 = 54 + 5 = 59$, $u_5 = 2 \times 59 + 5 = 118 + 5 = 123$, $u_6 = 2 \times 123 + 5 = 246 + 5 = 251$.

71. Evaluate: $\sum_2^{100} \left(\frac{1}{n-1} - \frac{1}{n}\right)$

$\sum_2^{100} \left(\frac{1}{n-1} - \frac{1}{n}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{98} - \frac{1}{99}\right) + \left(\frac{1}{99} - \frac{1}{100}\right) = \frac{1}{1} - \frac{1}{100} = \frac{99}{100}$

72. What is the sum of the terms of an infinite geometric sequence with first term 24 and common ratio $\frac{1}{4}$?

$S = \frac{24}{1-\frac{1}{4}} = \frac{24}{\frac{3}{4}} = 24 \times \frac{4}{3} = 8 \times 4 = 32$

73. What is the sum of the 15 smallest positive odd numbers?

This is a handy facto to know… $15^2 = 225$.

74. What is the sum of the even numbers between 21 and 59?

We're adding 22 through 58. Using outer pairs, each pair will add up to $22 + 58 = 80$. There are $\frac{58-22}{2} + 1 = \frac{36}{2} + 1 = 18 + 1 = 19$ numbers in our list, for a sum of $\frac{19}{2} \times 80 = 19 \times 40 = 760$.

75. One marble is drawn from a bag containing 3 red marbles, 5 white marbles, and 7 blue marbles. What is the probability that the marble drawn is white?

$\frac{5}{3+5+7} = \frac{5}{15} = \frac{1}{3}$

76. One card is drawn from a standard 52-card deck. What is the probability that the card drawn is either a king or a heart (or both)?

There are 13 hearts, and three other kings, for a probability of $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$.

77. When four fair coins are flipped, what is the probability that exactly one of them displays heads?

There are $2^4 = 16$ ways for four coins to flip. $4c1 = 4$ of these will have exactly one head, for a probability of $\frac{4}{16} = \frac{1}{4}$.

78. When two standard six-sided dice are rolled, what is the probability that the sum of the numbers shown is four?

There are $6^2 = 36$ ways for two dice to roll, and the ways to get a sum of 4 are 1&3, 2&2, and 3&1 (which is different), for a probability of $\frac{3}{36} = \frac{1}{12}$.

79. Evaluate: $\binom{7}{3}$

$7c3 = \frac{7!}{3! \times 4!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} = \frac{7 \times 6 \times 5}{3 \times 2} = 7 \times 5 = 35$

80. When five people sit at a round table, how many relative arrangements are possible?

Where the first person sits doesn't matter. After that, there are four people who could sit to her right, then three people that could sit beyond him, then two people for the next seat, and finally just one person who must take the last spot. Thus, our answer is $4 \times 3 \times 2 \times 1 = 24$.

81. Ralph and Thea plan to meet at the game shop sometime between 4 PM and 5 PM. If each shows up at a random time in that interval, waits up to 30 minutes for the other, and then leaves if the other one hasn't shown up, what is the probability that they will meet as planned?

This is a continuous probability problem, rather than a discrete one, and so geometry is a reasonable way to analyze it. Consider a square with edges of 60, where the left edge represents when Ralph arrives and the bottom edge represents when Thea arrives. If Ralph arrives at 4:00, then Thea can arrive any time between 4:00 and 4:30 and they'll meet. As Ralph arrives later, Thea's latest good arrival time gets later as well, until if Ralph arrives at 4:30, it doesn't matter when Thea arrives. The bottom half of our square has a bad triangle in the lower right now. Similarly, a bad triangle will form in the upper left as we continue this analysis. So, the total area of the square is $60^2 = 3600$, and the bad area is $30^2 = 900$, so the probability that they meet is $\frac{3600-900}{3600} = \frac{2700}{3600} = \frac{27}{36} = \frac{3}{4}$.

82. On the first day of school, Mrs. Tessandore has five identical pencils to distribute among her three students. If fairness doesn't matter to her, how many different distributions of pencils are possible?

Three pencils could go 5-0-0 in three ways, 4-1-0 in six ways, 3-2-0 in six ways, 3-1-1 in three ways, and 2-2-1 in three ways, for a total of $3 + 6 + 6 + 3 + 3 = 21$ ways.

83. What is the median of the data set {589, 78, 9345, 7, 894}?

Eliminating highs and lows gets rid of 9245 & 7, then 894 & 78, leaving 589.

84. What is the population standard deviation of the data set {0, 4}?

The standard deviation is the root of the mean of the squares of the deviations. The mean is $\frac{0+4}{2} = \frac{4}{2} = 2$, so the deviations are -2 and 2, their squares are 4 and 4, their mean is 4, and its root is 2.

85. Set L is the set of positive one-digit composite integers and Set S is the set of positive one-digit integers. How many supersets of Set L are subsets of Set S?

S has nine elements, from 1 to 9. L is 4, 6, 8, and 9. For a superset of L to be a subset of S, each of 1, 2, 3, 5, and 7 can choose whether or not to join, so there are $2^5 = 32$ such sets.

86. If $\boldsymbol{m} =< \mathbf{4}, -\mathbf{1} >$ and $\boldsymbol{r} =< \mathbf{1}, \mathbf{3} >$, evaluate $\mathbf{2}\boldsymbol{r} - \mathbf{3}\boldsymbol{m}$.

$2r - 3m =< 2,6 > -< 12, -3 >=< -10,9 >$

87. What is the area of a triangle with vertices at the points $(2,5)$, $(-3,2)$, and $(-1,5)$?

This triangle has a base of $2-(-1)=3$ and a height of $5-2=3$ for an area of $\frac{3\times 3}{2}=\frac{9}{2}$.

88. In the cryptarithm below, each instance of a given letter represents the same digit (0-9), and different letters represent different digits. What is the largest possible value of the four-digit positive integer $ABCD$?

$$\begin{array}{r} AB \\ \underline{-C} \\ D \end{array}$$

Obviously, $A=1$. To get a large $ABCD$, we'd like a large B, then C, then D. B will have to be less than C for the carrying to work properly, so B might be 8 if C is 9, except that this would require D to also be 9. If B is 7 and C is 9, then D can be 8, for an answer of 1798.

89. In the cross-number puzzle to the right, A, B, C, and D are different digits that together satisfy the four equations (two across and two down). What is the product of A, B, C, and D?

A	x	B	=	12
x		+		
C	+	D	=	12
=		=		
18		9		

Looking sideways, A could be 2, 3, 4, or 6, with B being the reverse. Looking down, A could be 2, 3, 6, or 9, with C being the reverse (and now A can't be 4 or 9). Looking down, D could be 3, 5, or 7. Looking sideways, D could be 3 or 6. Thus, D is 3, C is 9, A is 2, and B is 6, for an answer of $2\times 3\times 6\times 9=36\times 9=324$.

90. Evaluate: $\frac{4}{4-\frac{4}{4-\frac{4}{4-\frac{4}{\ddots}}}}$

Let $x=\frac{4}{4-\frac{4}{4-\frac{4}{4-\frac{4}{\ddots}}}}=\frac{4}{4-x}$. Cross-multiplying gives $4x-x^2=4$, which can be restructured to be $x^2-4x+4=0$, which factors to $(x-2)^2=0$ with a single root at $x=2$.

91. If $\cos p=\frac{4}{5}$ and $0<p<\pi$, evaluate $\sin p$.

Because of the quadrant, the sine will be positive, so $\sin p=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$.

92. What is the area, in square meters, of a triangle with sides measuring 3 m, 5 m, and 6 m?

Heron's formula gives $A=\sqrt{7\times 1\times 2\times 4}=2\sqrt{14}$.

93. Evaluate: $\cot\frac{5\pi}{3}$

$\frac{5\pi}{3}$ is in the fourth quadrant, so the cotangent will be negative. The reference angle is $\frac{\pi}{3} = 60°$, so the tangent would be $\sqrt{3}$, so our answer is $-\frac{\sqrt{3}}{3}$.

94. What are the rectangular coordinates, in the form (x, y, z), of the cylindrical coordinates $\left(6, \frac{2\pi}{3}, -1\right)$?

$x = r\cos\theta = 6\cos\frac{2\pi}{3} = 6\left(-\frac{1}{2}\right) = -3$, $y = r\sin\theta = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$, and $z = z = -1$.

95. What is the remainder when $b^5 - 4b^3 + 2b - 9$ is divided by $b + 2$?

Substituting the root $b = -2$ gives $(-2)^5 - 4(-2)^3 + 2(-2) - 9 = -32 + 32 - 4 - 9 = -13$.

96. Evaluate: $\lim_{c\to 3^-}\lfloor c\rfloor$

For numbers *just* less than 3, the floor function consistently produces 2, so that is the limit.

97. Evaluate: $\lim_{w\to\infty}\frac{5w^2-4w+3}{2w^2+9}$

For large values of w, w^2 is *way* bigger than any number of w's, and *way*, *way* bigger than constants, so we can ignore those terms, getting $\frac{5w^2}{2w^2} = \frac{5}{2}$.

98. If $d(v) = 2(\ln v)^3$, evaluate $d'(e)$.

$d'(v) = 3 \times 2(\ln v)^2\left(\frac{1}{v}\right)$, so $d'(e) = 6(\ln e)^2\left(\frac{1}{e}\right) = \frac{6\times 1^2}{e} = \frac{6}{e}$.

99. If $u(f) = 3\pi^2 + 1$, evaluate $u'(4)$.

There are no variables in $u(f)$, and specifically no f, so $u'(f) = 0$ and specifically $u'(4) = 0$.

100. What is the area of the region bounded by $y = x^2$ and $y = x^3$?

This will be $\int_0^1 (x^2 - x^3)dx = \left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.