2014 Fall Startup Event
Thursday, September 25th, 2014

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. What is the difference between the smallest positive four-digit integer and the largest two-digit integer?
2. Evaluate: 245×97
3. Evaluate: $4002 \div 46$
4. What is the sum of 251, 25.1, and 2.51? Express your answer as a **decimal**.
5. 48 is 60% of what number?
6. Evaluate: $-(-1)(-2) - (-3) \div (-4)$
7. How many seconds are there in three and a half hours?
8. Evaluate: $\left(\frac{2}{3}\right)^4$
9. Evaluate: $9 \times 8^2 \div (7 + 6 - 5)$
10. Express in simplest radical form: $\sqrt{525}$
11. Evaluate: $1001^2 - 999^2$
12. What value(s) of *z* satisfy $9 - 8z = 57$?
13. What value(s) of *y* satisfy $2(3 - y) = 4(5y + 6)$?
14. What value(s) of *x* satisfy $x^2 - 6x + 8 = 0$?
15. Normally, I can fill my bathtub in 8 minutes and drain it in 12 minutes, but today I accidentally left the drain open when I turned on the water to fill it. How many minutes did it take my tub to fill?
16. If three spigots can fill five buckets in four minutes, how many spigots would be necessary to fill ten buckets in one minute?
17. If ten liters of pure acid are mixed with five liters of a solution that is 10% acid, what percent of the resulting solution will be acid?
18. If two numbers have a sum of 84 and a difference of 36, what is the smaller number?

19. What are the coordinates of the *x*-intercept of the line $3x - 4y = 24$?

20. What is the slope of a line perpendicular to the line $3x + 4y = 24$?

21. What is the distance between the points $(4, -7)$ and $(-2,3)$?

22. What is the shortest distance from the point $(1,2)$ to the line $x + y = -2$?

23. Which quadrant does the line $y = 3x + 1$ NOT pass through?

24. What is the equation of the axis of symmetry of $y = 2x^2 - 28x + 82$?

25. What are the coordinates, in the form (x, y), of the vertex of $y = x^2 + 6x - 11$?

26. What is the smallest positive two-digit integer that increases by 72 when its digits are reversed?

27. Roger's fenced yard measures 20 m by 30 m. He paves a path one meter wide all around the perimeter of his yard (touching the inside of the fence). What is the area, in square meters, of the path?

28. If you can buy 20 kilograms of corn meal for D dollars, how many cents would it take to buy K kilograms of corn meal?

29. If 3 Wombats can be exchanged for 4 Vultures and 10 Vultures can be exchanged for 21 Slugs, how many Slugs could you get with 120 Wombats?

30. What value(s) of q satisfy $\frac{q}{1+q} = \frac{2-q}{5-q}$?

31. Paul is currently three times Olive's age, but in four years he will be twice her age. How old is Paul?

32. Simplify by multiplying and combining like terms: $(n + 3)(2 - 5n)$

33. What is the area, in square meters, of a right triangle with legs measuring 3 m and 4 m?

34. What is the perimeter, in meters, of an equilateral triangle with sides measuring 5 m?

35. What is the area, in square meters, of a triangle with sides measuring 6 m, 7 m, and 7 m?

36. What is the most specific name that applies to all triangles with exactly one angle greater than 90°?

37. What is the most specific name that applies to all polygons with exactly four sides?

38. What is the perimeter, in meters, of a parallelogram with sides measuring 8 m and 9 m?

39. What is the area, in square meters, of a circle with a radius of 10 m?

2014 Fall Startup Event
Thursday, September 25th, 2014

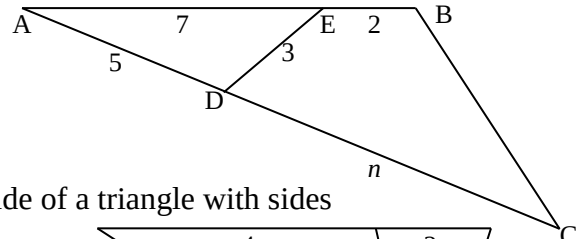
40. What is the volume, in cubic meters, of a right square pyramid with base edges measuring 9 m and a height of 8 m?

41. What is the name for a polygon with seven sides?

42. Two similar pentagons have areas of 40 m^2 and 90 m^2. If the smaller pentagon has a perimeter of 30 m, what is the perimeter, in meters, of the larger pentagon?

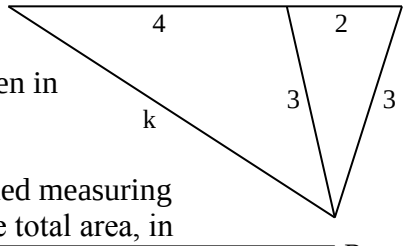
43. What is the surface area, in square meters, of a cube with a volume of 512 m^3?

44. In the figure to the right, $m\angle AED = m\angle BCA$, and all segment lengths are given in meters. What is the value of n?



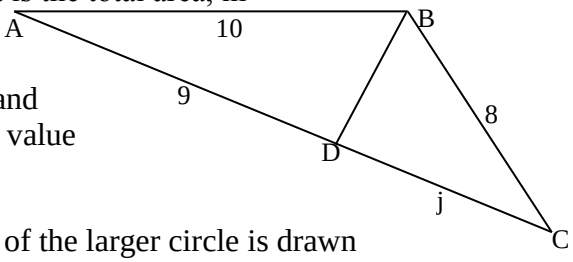
45. What is the altitude, in meters, to the longest side of a triangle with sides measuring 6 m, 9 m, and 7 m?

46. In the figure to the right, with all segment lengths given in meters, what is the value of k?



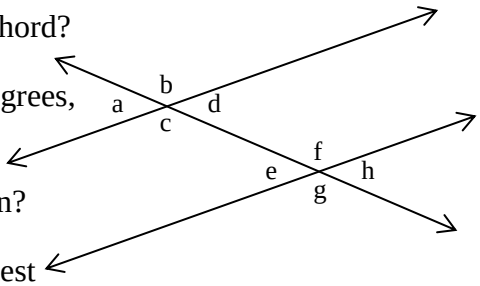
47. A goat is tied to an external corner of a rectangular shed measuring 4 m by 6 m. If the goat's rope is 8 m long, what is the total area, in square meters, in which the goat can graze?

48. In the figure to the right, where $\overline{BD}$ bisects $\angle B$ and segment lengths are given in meters, what is the value of j?



49. When two concentric circles are drawn, a chord of the larger circle is drawn tangent to the smaller circle. If the area of the annulus between the two circles is 49π m^2, what is the length, in meters, of the chord?

50. In the figure to the right, with all angles measured in degrees, if $a = 57$, what is $f + g + h$?



51. How many diagonals can be drawn in a regular nonagon?

52. When three lines are drawn in a plane, what is the greatest number of regions into which they can divide the plane?

53. What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 6:10 AM?

54. If a solid blue cube is painted red and then cut into 216 smaller cubes, how many of the smaller cubes will have some red paint on them?

55. Simplify in terms of i $\left(i = \sqrt{-1}\right)$: $i(2 - 3i)(4i + 5)$

2014 Fall Startup Event
Thursday, September 25th, 2014

56. Simplify in terms of i $\left(i = \sqrt{-1}\right)$: $2i^3 + 5i^6 - 9i^{10}$

57. What are the coordinates, in the form (x, y), of the center of the ellipse $x^2 + 4y^2 - 6x + 4y - 100 = 0$?

58. Evaluate: $\log_2 256$

59. What is the smallest integer value of h for which $g(h) = 9 + 8\,(7^{h-6})$ is greater than 1000?

60. If f is directly proportional to g and $f = 24$ when $g = 6$, what will f be when $g = 18$?

61. If $d(c) = 3c\sqrt{30 + c - c^2}$ has a domain and range that are both subsets of the real numbers, express the domain in interval notation.

62. If Tomium has a half-life of 20 seconds, how many **grams** of a 4096 kg sample will remain after five minutes?

63. When $(2a - 3b)^{45}$ is expanded and like terms are combined, how many terms will there be?

64. Evaluate: $64^{-\frac{5}{2}}$

65. Express the base six numeral 123_6 as a base ten numeral.

66. Express the base ten numeral 234_{10} as a base four numeral.

67. Express the sum of the base seven numerals 456_7 and 542_7 as a base seven numeral.

68. What is the prime factorization, in exponential form, of 684?

69. What is the sum of the positive integer factors of 440?

70. How many multiples of 8 are factors of 1200?

71. What is the least common multiple of 24 and 45?

72. How many positive five-digit integers contain at least one odd digit?

73. When 12^{15} is evaluated, what is the units digit?

74. What is the sixth term of a geometric sequence with a first term of 11 and a common ratio of 2?

75. What is the missing term of the sequence 7, 10, 15, 22, 31, __, 55, 70, 87, …?

76. What is the fourth term of a harmonic sequence with a first term of $\frac{1}{2}$ and a second term of $\frac{1}{5}$?

77. What is the sum of the counting numbers less than 80?

2014 Fall Startup Event
Thursday, September 25th, 2014

78. What is the sum of the twelve smallest odd counting numbers?

79. What is the sum of the eight smallest positive perfect cubes?

80. When you draw a single card from a standard 52-card deck, what is the probability that it is a red card or a face card (or both)?

81. When you flip five coins, what is the probability that exactly three of them are heads?

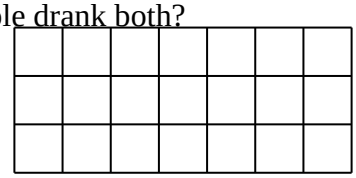
82. When two fair six-sided dice are rolled, what is the probability that the numbers showing sum to nine?

83. Evaluate: ${}_{12}C_5$

84. I have four Huey Lewis CDs and five Celine Dion CDs that I keep next to one another on a shelf. If I keep each group of CDs together, how many total arrangements of the CDs are possible?

85. At my party, 25 people drank Poke and 16 people drank Cepsi. If there were 36 people at the party and 9 people drank neither Poke nor Cepsi, how many people drank both?

86. In the grid of unit squares to the right, how many paths of length ten are there from the upper left corner to the lower right corner?



87. In how many ways can I distribute ten identical breadcrumbs to four ducks if I don't necessarily care about fairness?

88. Evaluate: $< 9, -3 > - < 1,2 >$

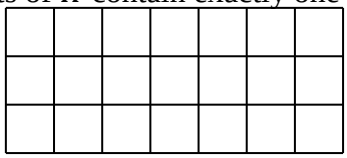
89. What value of w will make the matrix $\begin{bmatrix} 4 & 3 \\ 2 & w \end{bmatrix}$ singular?

90. What is the median of the data set $\{3, 9, 2, 14, 7, 6, 9, 7, 4, 9, 6\}$?

91. If Set V is the set of positive two-digit multiples of six and Set U is the set of counting numbers with at least one odd digit, how many elements are in the set $V \cap U^c$?

92. Let K be the set of prime numbers less than 20. How many subsets of K contain exactly one instance of the digit 1?

93. How many squares of any size are there in the grid of unit squares to the right?



94. Write an expression that evaluates to 2 by using the digits 2, 3, and 4 exactly once each, and the operations of addition, subtraction, multiplication, and division (and parentheses) as many times as you like.

95. A right triangle has legs measuring 3 m and 4 m. What is the cotangent of the smallest angle?

96. An angle in the third quadrant has a sine of $-\frac{1}{4}$. What is the cosine of this angle?

97. If $\cos r = \frac{1}{3}$, what is the smallest possible value of $\sin(2r)$?

98. Evaluate: $\lim_{r\to\infty} \frac{2r^3+4r^5}{6r^7-8r}$

99. If $q(p) = 5\sqrt{7p+9}$, evaluate $q'(0)$.

100. Evaluate: $\int_2^4 \left(6x + \frac{8}{x}\right) dx$