

2016 Team Scramble
Thursday, November 3rd, 2016

Easier Problems

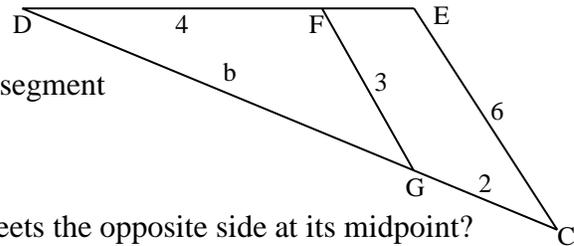
1. Evaluate: $8065 + 63857$
2. Evaluate: $1394940 \div 804$
3. How many minutes are in three days?
4. Evaluate: $\frac{496}{7} \div \frac{24}{56}$
5. Express 34567.04 in scientific notation rounded to four significant figures.
6. Evaluate: $\frac{14! \cdot 9! \cdot 5!}{7! \cdot 11! \cdot 8!}$
7. Express in simplest radical form: $\sqrt{6804}$
8. Simplify by rationalizing the denominator: $\frac{156}{\sqrt{24} - \sqrt{11}}$
9. Write the variables in order of ascending value (e.g. BADC):
 $A = 946 + 3463$ $B = 5793 - 682$ $C = 19 \times 26$ $D = 67895 \div 13$
10. What value(s) of f satisfy $5f - 4 = 46 + 7f$?
11. What value(s) of g satisfy $6g^2 - g = 15$?
12. Nancy has 12 liters of a 45% acid solution that she wishes to strengthen to an 80% acid solution. If she can only do this by mixing it with a 90% acid solution, how many liters of the 90% acid solution should she add?
13. Pikachu and Charmander see one another at the same moment, from a distance of 1.5 km. If Pikachu runs away at 46 mps (meters per second) and Charmander chases him at 71 mps, how many seconds will it take Charmander to catch Pikachu?
14. In which quadrant does the point $(-735951, 379347)$ lie?
15. What are the coordinates, in the form (x, y) , of the intersection of the lines $y = 3x - 7$ and $2x - 3y = -7$?
16. What is the equation for the axis of symmetry of the parabola $y = 3x^2 + 9x - 25$?
17. What are the coordinates, in the form (x, y) , of the rightmost x-intercept of the parabola $y = 4x^2 + 3x - 10$?
18. Professor Plum wrote an equation of the form $0 = q^2 + Aq + B$ on the board for her students to solve. Violet used the wrong value of A and got roots of -1 and 4. Lavender used the wrong value of B and got roots of 1 and -10. What were the actual roots of the equation?

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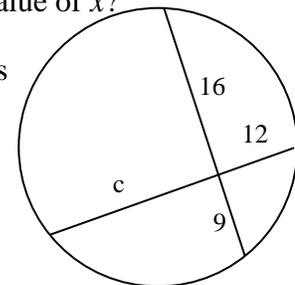
19. If you can buy P pianos for \$10,000, how many dimes would it take to buy 100 pianos?
20. A field contains llamas (four legs) and emus (two legs). If there are a total of 62 legs and 22 heads, how many llamas are there?
21. In the system of equations $4r + 6s - t = 67$ and $4t - 3s - 2r = 45$, what is the value of t ?
22. What is the solution, in the form (w, x) , of the system of equations $4w - 6x = -22$ and $7w + x = -4$?
23. I ran the two miles home from school in just 16 minutes, but I got driven to school along the same route at a speed of 30 miles per hour. What was my average speed, in miles per hour, for the round trip?

24. What are the coordinates, in the form (x, y) , when the point $(7, 9)$ is rotated 90° clockwise about the point $(-6, 2)$?
25. A right triangle with a 30° angle has a hypotenuse measuring 12 m. What is its area, in square meters?



26. In the $\triangle CDE$ to the right, $\overline{FG} \parallel \overline{CE}$ and all given segment lengths are in meters. What is the value of b ?
27. What is the name for a triangle cevian which meets the opposite side at its midpoint?
28. A pentagon has sides measuring 20 m, 1 m, 2 m, 3 m, and x m. What is the sum of the largest possible integer value of x and the smallest possible integer value of x ?

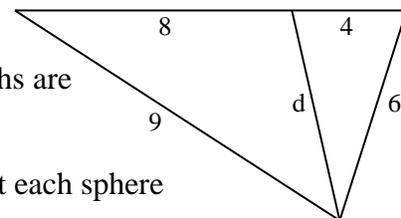
29. What is the area, in square meters, of a sector of a circle with a radius of 2 m and an arclength of 4 m?
30. Two chords intersect in the circle to the right, with all segment lengths given in meters. What is the value of c ?



31. What is the circumference, in meters, of a circle inscribed inside a regular hexagon with an area of $12\sqrt{3} \text{ m}^2$?

32. What is the name for a solid with nine planar faces?

33. In the triangle to the right with one cevian, all segment lengths are given in meters. What is the value of d ?



34. What is the largest number of spheres you can have such that each sphere touches every other sphere?
35. Two concentric circles have radii that differ by 10 m. If the area between the two circles is $100\pi \text{ m}^2$, what is the length of a chord of the outer circle that is tangent to the inner circle?

2016 Team Scramble
Thursday, November 3rd, 2016

36. What are the coordinates, in the form (x, y) , of the center of the conic section $4x^2 + 5y^2 + 24x - 40y = 0$?
37. What value(s) of w satisfy $3^{2w+1} - 28 \cdot 3^w + 9 = 0$?
38. If $v(u) = 3u^2(u + 1)$ and $t(s) = \frac{s}{s^2+1}$, what is the value of $v(t(-2))$?
39. The half-life of Cerium is twelve minutes. How many **grams** of a 9000 kg sample of Cerium will remain after an hour?
40. Express the base ten numeral 624_{10} as a base eleven numeral.
41. Express the difference $4826_9 - 1356_9$ as a base nine numeral.
42. What is the largest number less than 100 that leaves a remainder of 1 when it is divided by three and a remainder of 3 when it is divided by 4?
43. What is the 268th term of an arithmetic sequence with first term 417 and common difference 34?
44. Evaluate: $\prod_{q=1}^{10} \frac{q+1}{q+3}$
45. What is the sum of the first 9 terms of a geometric sequence with first term 120 and common difference ratio $\frac{1}{2}$?
46. What is the mode of the data set $\{1, 2, 6, 7, 8, 0, 4, 1, 3, 8, 7, 8, 0, 1, 7, 8, 0\}$?
47. When two cards are drawn from a standard 52-card deck, what is the probability that the first one has a lower rank than the second one?
48. When three fair six-sided dice are rolled, what is the probability that exactly two of them show the same number?
49. What is the shortest distance from the point $(-4, 1, -3)$ to the plane $3x - 4y - 5z = 6$?
50. How many subsets of $\{7, 45, 8, 9, 14, 5, 79\}$ are supersets of $\{8, 45, 9\}$?
51. Express the spherical coordinates $(4, \frac{3\pi}{2}, \frac{5\pi}{6})$ as polar coordinates. The spherical coordinates list the radius, azimuthal angle, and polar angle in that order.
52. Evaluate: $\cot\left(-\frac{57\pi}{6}\right)$
53. Evaluate: $\lim_{n \rightarrow 4} \frac{3n^3 - 14n^2 + 9n - 4}{4 - n}$
54. Evaluate: $\lim_{n \rightarrow e} \frac{e^n - e^e}{n - e}$

2016 Team Scramble
Thursday, November 3rd, 2016

55. What are the coordinates, in the form (x, y) of the leftmost critical point of $y = 2x^3 - 3x^2 - 12x - 5$?

Harder Problems

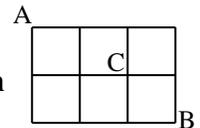
56. Evaluate: 57.9×34.16
57. What percent of 32 is 76?
58. Evaluate: $121^3 - 119^3$
59. What is the solution, in the form (j, k, m) , of the system of equations $j + k + m = -6$, $j - k - 2m = 4$, and $-3j + 2k + m = 7$?
60. Jack could build the brick wall in twelve hours and Jill could build it in ten hours. They're asked to work on the wall together, but because they talk to each other, they lay 100 fewer total bricks per hour than they would have if they were working separately, and thus it takes them seven hours to complete the wall. How many bricks were in the wall, to the nearest brick?
61. The IB group decided to order the World's Best Pizza and split the cost evenly. If there had been one more member, each person would have paid \$.40 less. If there had been one fewer member, each person would have paid \$.50 more. How many people are in the IB group?
62. What is a solution, in the form (u, v, w) , of the system of equations $u + v + w = 12$, $uv + vw + uw = -43$, and $uvw = 30$?
63. Xerxes was 12 when Yolanda was twice Zed's age, and Zed was 3 when Yolanda was twice Xerxes' age. If they all share the same birthday in different years, how old will Yolanda be when Zed is 30?
64. A jaguar at position $(5,3)$ wishes to drink from the stream $(x - y = 8)$, then return to the tree where she stored a gazelle's body at position $(-2,1)$. What is the shortest distance she can travel?
65. A professor computes the average of her students' scores on a recent test, getting a value of 70. However, she realized that although she had divided by the correct **number** of student scores, she had forgotten to include one test score when she computed the **total** of the scores. She adds the missing score, recalculating the total correctly, but absent-mindedly also adds one to the number of scores, getting a new incorrect average of 71. Realizing she's made another oversight, she correctly calculates the average to be 72. What is the lowest possible value of the missing test score that started all this madness?
66. A right triangle has an area of 84 m^2 and a perimeter of 56 m. What is the length, in meters, of its hypotenuse?

2016 Team Scramble
Thursday, November 3rd, 2016

67. What is the smallest possible perimeter, in meters, of a rectangle with an area of 1485 m^2 and integer side lengths when measured in meters?
68. Two circles have radii of 41 m and 18 m, and have their centers 295 m apart. What is the length in meters of one of their common internal tangents?
69. On a tessellated plane, every vertex is surrounded by a combination of squares and equilateral triangles. If no two squares share a side, what fraction of the plane is covered by squares? Note: only one size of square and one size of equilateral triangle are used.
70. What is the first time after 3:15:00 AM that the hour and minute hands of a standard 12-hour analog clock form the same angle as they did at 3:15? Express your answer to the nearest second.
71. Line segments are drawn from a triangle's incenter to each of its vertices, dividing it into areas of 10 m^2 , 17 m^2 , and 21 m^2 . What is the perimeter, in meters, of the triangle?
72. At how many points do the graphs of $y = \log_3 \frac{x}{4}$ and $\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = 9$ intersect?
73. At the pizza buffet, Sam can eat two slices of pizza every minute until there are no more slices available, at which point he leaves, even if another pizza appears at the instant he is leaving. The restaurant always slices their pizzas into eight slices, and brings out a whole pizza every M minutes. Strangely, there are never any other diners when Sam is at the buffet. Two weeks ago, there were four pizzas on the buffet at the moment he arrived, and he left after 20 minutes. Last week, there were six pizzas on the buffet at the moment he arrived, and he left after 36 minutes. What is the sum of all possible integer values of M ?
74. If $\log_2 3 = n$ and $\log_5 2 = p$, express $\log 9$ in terms of n and p .
75. What is the maximum value of the expression $|g - h| + |g - j| + |k - g|$ if $0 \leq k \leq j \leq h \leq g \leq 10$?
76. What is the sum of the reciprocals of the squares of the roots of $2r^3 - 5r^2 - 3r + 4 = 0$?
77. What is the minimum value of the expression $-\left\lfloor \frac{x}{2} \right\rfloor + \left\lfloor \frac{x}{3} \right\rfloor + \left\lfloor \frac{x}{5} \right\rfloor - \left\lfloor \frac{x}{7} \right\rfloor$ for $0 \leq x \leq 100$?
78. How many positive integers are factors of 4752 and multiples of 18?
79. The floor of a room is in the shape of a parallelogram with sides measuring 3 m and 6 m and corner angles of 120° and 60° . The room is tiled with a tessellation of equilateral triangles that each measure 50 cm on each side. As part of a remodel, a contractor draws the longer diagonal of this parallelogram on the floor. How many tiles does that line intersect? Note: intersecting a tile ONLY at a vertex does NOT count as intersecting the tile.

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80. Positive integers are all written in a line. Once this is done, the number 672 is colored blue, as is every 150th number afterward (833, 983, etc.). Once this is done, the number 264 is colored red, as is every 480th number afterward (744, 1224, etc.). What is the fewest numbers that appear between a red number and a blue number?
81. How many seven-digit palindromes are even and contain at most two different digits?
82. What is the twentieth term of the sequence beginning 8, 12, 19, 29, 42, ...?
83. What is the next term of the harmonic sequence 630, 504, 420, ...?
84. What is the missing term of the sequence 542, 582, 662, ____, 818, 882, 1010, ...?
85. A sequence has first term $m_1 = 11$ and subsequent terms $m_n = 3m_{n-1} - 3^n$. What is the fifth term of this sequence?
86. A point is chosen randomly on a yardstick, and the yardstick is broken at that point. Then a point is chosen randomly on the larger piece, and that piece is broken at that point. What is the probability that the shortest of the three pieces is shorter than 6 inches?
87. Two players take turns drawing (and keeping) a single marble from a bag that initially contains three green marbles and three white marbles. The winner is the first player to get two marbles that are the same color. What is the probability that the first person wins the game?
88. In D&D 5e, a player with “advantage” rolls two 20-sided dice and uses the higher of the two numbers shown. What is the expected value of the higher of the two numbers?
89. In the grid of unit squares to the right, how many paths of length 7 are there from A to B that pass through C at least once? Note: it is not okay for a path to pass through vertices and/or segments multiple times.



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93. In the cryptarithm below, each instance of a letter represents the same digit (0-9), and different letters represent different digits (e.g. if one A is a 1, all A's are 1's and B's cannot be 1's). What is the smallest possible value of the six-digit number ABCDEF?

$$\begin{array}{r} AB \\ \times CD \\ \hline BBB \\ DDDE \\ \hline AFEB \end{array}$$

94. An ant is on the face of a cube with edges measuring 6 m. The ant is currently at Point A, which is 1 m from one edge and 2 m from another edge. The ant wishes to travel to Point B, which is also 1 m from one edge and 2 m from another edge (there are many possible locations of this point). The ant will walk the shortest distance possible on the exterior of the cube to accomplish this. What is the longest distance, in meters, the ant could have to travel?
95. When five student line up for recess, Katie is exactly two places ahead of Isabella, Jesus is behind Li, Henri and Isabella are immediately adjacent (in either order), and Li is either first or last in line. If Jesus is behind Henri, write the first letters of each person's name in order from the front of the line to the back of the line.
96. A triangle has sides measuring 18 m and 24 m, with a 120° angle between them. What is the length, in meters, of the third side?
97. What is the area, in square meters, of a triangle with sides measuring 5 m, $3\sqrt{5}$ m, and $2\sqrt{10}$ m?
98. In how many points do $y = 9 \cos 8\pi x$ and $y = 10 - e^{11x}$ intersect?
99. I'm working on a 20-foot ladder leaning against a vertical wall and standing on a level floor. Suddenly the foot of the ladder begins to slip away from the wall at a speed of 8 meters per second! At the moment when the foot of the ladder is 16 feet from the wall, how fast is the top of the ladder sliding down the wall?
100. Consider the area bounded by $y = (x + 1)^2 + 3$ and $y = 7$. What is the volume passed through by this area when it is rotated around the line $x = 4$?